

## DUST TRANSPORT IN MRI TURBULENT DISKS: IDEAL AND NON-IDEAL MHD WITH AMBIPOLAR DIFFUSION

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## ABSTRACT

We study dust transport in turbulent protoplanetary disks using three-dimensional global unstratified magnetohydrodynamic (MHD) simulations including Lagrangian dust particles. The turbulence is driven by the magnetorotational instability (MRI) with either ideal or non-ideal MHD that includes ambipolar diffusion (AD). In ideal MHD simulations, the surface density evolution (except for dust that drifts fastest), turbulent diffusion, and vertical scale height of dust can all be reproduced by simple one-dimensional and/or analytical models. However, in AD dominated simulations which simulate protoplanetary disks beyond 10s of AU, the vertical scale height of dust is larger than previously predicted. To understand this anomaly in more detail, we carry out both unstratified and stratified local shearing box simulations with Lagrangian particles, and find that turbulence in AD dominated disks has very different properties (e.g., temporal autocorrelation functions and power spectra) than turbulence in ideal MHD disks, which leads to quite different particle diffusion efficiency. For example, MRI turbulence with AD has a longer correlation time for the vertical velocity, which causes significant vertical particle diffusion and large dust scale height. In ideal MHD the Schmidt numbers ( $Sc$ ) for radial and vertical turbulent diffusion are  $Sc_r \sim 1$  and  $Sc_z \gtrsim 3$ , but in the AD dominated regime both  $Sc_r$  and  $Sc_z$  are  $\lesssim 1$ . Particle concentration in pressure bumps induced by MRI turbulence has also been studied. Since non-ideal MHD effects dominate most regions in protoplanetary disks, our study suggests that modeling dust transport in turbulence driven by MRI with non-ideal MHD effects is important for understanding dust transport in realistic protoplanetary disks.

*Subject headings:* accretion, accretion disks - astroparticle physics - diffusion - dynamo - magnetohydrodynamics (MHD) - instabilities - turbulence - protoplanetary disks - meteorites, meteors, meteoroids - stars: pre-main sequence - stars: protostars

## 1. INTRODUCTION

Protoplanetary disks are probably turbulent (Hughes et al. 2011). Turbulence leads to mass accretion and builds the central star within the disk's lifetime (Hartmann et al. 1998).

Understanding solid transport in turbulent disks is important for interpreting the meteoritic record of our solar system (see Cuzzi & Weidenschilling 2006 for a review). Significant radial transport and mixing of solids may be necessary to explain the presence of Calcium-Aluminum-rich Inclusions (CAI) in chondritic meteorites (e.g., Cuzzi et al. 2003; Ciesla 2010), and the diversity of chondrites (e.g., Anders 1964; Zanda et al. 2006; Jacquet et al. 2012). Radial transport of solids has also been incorporated in various protoplanetary disk time-dependent models to understand the redistribution of solids in our solar nebulae and the implications for comets and meteorites (Cassen 1996, 2001; Gail 2001, 2002; Wehrstedt & Gail 2002; Bockele-Morvan et al. 2002; Hughes & Armitage 2010; Jacquet & Robert 2013).

Solid transport in turbulent disks is also crucial for planet and planetesimal formation (see Johansen et al. 2014 for a review). In the direction perpendicular to the disk midplane, dust settling towards the midplane

is balanced by turbulent diffusion, which determines the thickness of the dust disk (Weidenschilling 1980; Cuzzi et al. 1993; Carballido et al. 2006). When the dust disk is thin enough, it can lead to the gravitational collapse of solids into planetesimals (Safronov 1969; Goldreich & Ward 1973; Youdin & Shu 2002; Ward 2000; Youdin 2005a, 2005b).

Dust transport also has important implications for protoplanetary disk observations. Dust vertical settling in disks helps to explain the spectral energy distributions of protoplanetary disks (D'Alessio et al. 2006; Furlan et al. 2006; Pinte et al. 2008). The radial drift of dust relative to the gas could explain the dramatically different disk structures revealed by recent near-infrared (near-IR) polarization imaging and submillimeter (submm) interferometry (Dong et al. 2012; Zhu et al. 2012; Follette et al. 2013; Andrews et al. 2012; Qi et al. 2013).

Theoretical models on dust transport in turbulence have been developed over the years (Voelk et al. 1980; Markiewicz et al. 1991). The turbulent diffusion coefficient for dust particles was derived to understand the thickness of dust layers (Cuzzi et al. 1993; Dubrulle et al. 1995; Schräpler & Henning 2004; Carballido et al. 2006). To study particle collision, coagulation, and fragmentation, a model to derive relative velocities between colliding particles has also been developed (Cuzzi & Hogan 2003; Ormel & Cuzzi 2007; Carballido et al. 2008). A more refined model on particle turbulent diffusion including particle orbital dynamics in Keplerian disks (e.g., epicycles and vertical oscillations) has been provided by

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Youdin & Lithwick (2007).

Numerical simulations have also been carried out to study dust diffusion in disks with turbulence driven by the magnetorotational instability (MRI, Balbus & Hawley 1991). Using unstratified MHD shearing box simulations, Johansen & Klahr (2005) find that the vertical diffusion coefficient is lower than the turbulent viscosity (which is defined as the total  $r - \phi$  turbulent stress normalized by the local density and orbital frequency), while the radial diffusion coefficient is slightly larger than the turbulent viscosity. On the other hand, when strong net vertical magnetic fields are imposed, both the radial and vertical diffusion coefficients can be significantly smaller than the turbulent viscosity (Carballido et al. 2005; Johansen et al. 2006). The vertical settling of small and large particles in turbulent disks have been studied using both analytical methods and numerical simulations (Fromang & Papaloizou 2006; Carballido et al. 2006; Turner et al. 2010). The radial diffusion coefficients of dust particles in MRI turbulence have also been directly measured in simulations (Carballido et al. 2011), which confirm the analytical formulae suggested by Youdin & Lithwick (2007). Global simulations have also been carried out to study dust transport in global disks. Fromang & Nelson (2009) suggest that the diffusion coefficients are higher at larger  $z$  since the velocity fluctuations increase significantly at the disk upper layers. Particles can also be trapped in the large-scale disk structures induced by MHD turbulence (Lyra et al. 2008), such as vortices (Fromang & Nelson 2005) and zonal flows (Johansen et al. 2009).

However, almost all these previous numerical simulations assume ideal MHD. In a realistic protoplanetary disk, ideal MHD is a good approximation only within 0.1 AU where the ionization fraction is large. Non-ideal MHD effects (e.g., Ohmic dissipation, ambipolar diffusion, and the Hall effect) play essential roles in disks beyond 0.1 AU (see Armitage 2011; Turner et al. 2014 for a review). From 0.1 AU to several AU, a MRI “dead zone” due to Ohmic resistivity may exist (Gammie 1996). Dust settling in the “dead zone” has been explored by Fromang & Papaloizou (2006), Turner et al. (2010), and Okuzumi & Hirose (2011). In the outer disks beyond 10s of AU, ambipolar diffusion (AD) dominates (Bai 2011a, 2011b; Perez-Becker & Chiang 2011a, 2011b). Although the effects of AD on MRI turbulence have been studied both analytically (Blaes & Balbus 1994; Kunz & Balbus 2004) and through numerical simulations (Mac Low et al. 1995; Brandenburg et al. 1995; Hawley & Stone 1998), dust transport in such regions has not been explored yet. Since current millimeter observations (e.g. *CARMA*, *SMA*, *ALMA*, *EVLA*) are sensitive to dust in disks at 10s of AU, studying dust transport in turbulent disks dominated by AD is very important.

In this paper, we study dust transport in MRI turbulent disks in the AD dominated regime for the first time. We will show that the properties of turbulence induced by the MRI with AD can be dramatically different from turbulence induced by the MRI with ideal MHD, which has large effects on particle transport in protoplanetary disks.

By carrying out global simulations with Lagrangian particles, we also test if a simple one-dimensional (1-D) time-dependent model for the dust disk can reproduce

the evolution of dust in three-dimensional (3-D) simulations. If simple 1-D models can be justified, the evolution of dust can be studied in long timescales without the need of expensive 3-D MHD simulations (e.g. simple models in Birnstiel et al. 2013). These global MHD simulations also enable us to study particle trapping in the zonal flows.

In Section 2, the analytical theory of particle transport in turbulent disks is reviewed. In Section 3, we describe our numerical setup. Our results are presented in Section 4. In Section 5, shearing box simulations are carried out to understand the differences between particle diffusion in ideal and non-ideal MHD with AD. A short discussion is given in Section 5, and conclusions are drawn in Section 6.

## 2. THEORY ON PARTICLE TRANSPORT IN TURBULENCE

When the gas disk evolves, dust particles will not only follow the gas but also drift, diffuse, and settle in the gas disk. In the radial direction, the surface density of the dust follows the contaminant equation (Morfill & Voelk 1984)

$$\frac{\partial \Sigma_d}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} [r(F_{diff} + \Sigma_d v_{d,r})] = 0, \quad (1)$$

where  $F_{diff}$  is the radial mass flux of dust due to turbulent diffusion, and  $\Sigma_d v_{d,r}$  is the mass flux due to dust radial drift from gas-drag.  $F_{diff}$  can be written as

$$F_{diff} = -D_{d,r} \Sigma_g \frac{\partial}{\partial r} \left( \frac{\Sigma_d}{\Sigma_g} \right), \quad (2)$$

where  $D_{d,r}$  is the dust diffusion coefficient in the radial direction, and  $\Sigma_d$  and  $\Sigma_g$  are the dust and gas surface density. The dust radial velocity  $v_{d,r}$  due to gas-drag is

$$v_{d,r} = \frac{v_{g,r} T_s^{-1} - \eta v_K}{T_s + T_s^{-1}}, \quad (3)$$

where  $v_K$  is the Keplerian velocity, and  $\eta$  is the ratio between the pressure gradient and the gravitational force. In an unstratified disk, we have  $\eta = -(r\Omega^2 \Sigma_g)^{-1} \partial P / \partial r$ .  $T_s$  is the dimensionless form of the dust stopping time  $t_s$  ( $T_s \equiv t_s \Omega$ ). We can also incorporate the diffusion term of Equation (1) into the dust velocity so that the equivalent total dust velocity is

$$v_{d,tot} = \frac{v_{g,r} T_s^{-1} - \eta v_K}{T_s + T_s^{-1}} - \frac{D_{d,r}}{r} \frac{d \ln(\Sigma_d / \Sigma_g)}{d \ln r}, \quad (4)$$

where the last term is the velocity due to turbulent diffusion.

In the vertical direction, dust particles also settle in the disk following the equation of motion

$$\frac{dv_{d,z}}{dt} = -\Omega^2 z_d + \frac{v_{g,z} - v_{d,z}}{t_s}, \quad (5)$$

where the first term on the right is the vertical gravity toward the disk midplane, and the second term on the right is the acceleration due to gas drag. In this equation,  $v_{g,z}$  and  $v_{d,z}$  are vertical velocities of the gas and dust particles at the particles' vertical positions  $z_d$ . When the dust stopping time  $t_s \ll \Omega^{-1}$  and the gas disk is stationary ( $v_{g,z} = 0$ ), the particle reaches the terminal

velocity of  $v_{d,t} = -\Omega^2 t_s z_d$  before the particle falls to the disk midplane by gravity.

If the disk is turbulent ( $v_{g,z} \neq 0$ ), particles can be lifted off the disk midplane by turbulent diffusion, and the vertical structure of the dust disk can reach a steady state when the mass flux due to vertical settling balances the mass flux due to turbulent diffusion,

$$\rho_d v_{d,t} = D_{d,z} \rho_g \frac{\partial \rho_d}{\partial z} \frac{\rho_d}{\rho_g}, \quad (6)$$

where  $\rho_g$  and  $\rho_d$  are the gas and dust density along the  $z$  direction, and  $D_{d,z}$  is the dust diffusion coefficient in the vertical direction. When the gas disk has a Gaussian density profile as  $\rho_g(z) = \rho_{\text{mid},g} \exp(-z^2/2h^2)$  where  $h$  is the scale height of the gas disk  $h \equiv c_s/\Omega$ , Equation (6) can be solved to derive the vertical density profile of the dust as  $\rho_d(z) = \rho_{\text{mid},d} \exp(-z^2/2h_d^2)$  with a scale height of

$$h_d = \frac{h}{\sqrt{h^2 \Omega^2 t_s / D_{d,z} + 1}}. \quad (7)$$

If the gas is unstratified (e.g.,  $\rho_g$  is a constant along the  $z$  direction), as in our unstratified global and local simulations, we can solve Equation (6) and the dust in disks has a scale height of

$$h_d = \sqrt{\frac{D_{d,z}}{\Omega^2 t_s}}. \quad (8)$$

At this point, the evolution of dust is fully determined by the radial and vertical turbulent diffusion coefficients— $D_{d,r}$  and  $D_{d,z}$ —from Equations (1), (7) and (8). These turbulent coefficients play essential roles in the evolution of dust, similar to the important role of turbulent viscosity in the gas disk evolution.

The turbulent diffusion coefficients have been derived in various works (Volk et al. 1980; Markiewicz et al. 1991; Cuzzi et al. 1993; Dubrulle et al. 1995; Schrapler & Henning 2004; Carballido et al. 2006; Youdin & Lithwick 2007). Here, we adopt the formulation from Youdin & Lithwick (2007) who present a thorough theoretical model including the orbital dynamics of particles in Keplerian disks. To enable comparison with our simulations, we highlight some aspects of their theoretical model below.

First, we focus on the dust vertical diffusion coefficient  $D_{d,z}$  which only involves particle motion in the  $z$  direction, and it is defined as

$$D_{d,z} \equiv \frac{1}{2} \frac{d\langle z_d^2 \rangle}{dt}. \quad (9)$$

The ensemble average can be taken by using trajectories of either many particles over a short period of time, or one particle over a long time. When dust is very small and couples with the gas almost perfectly ( $t_s \ll \Omega^{-1}$ ),  $D_{d,z}$  also equals the gas diffusion coefficient  $D_{g,z}$ ,

$$D_{g,z} = \frac{1}{2} \frac{d\langle z_g^2 \rangle}{dt} = \int_0^\infty \langle v_{g,z}(\tau) v_{g,z}(0) \rangle d\tau = \int_0^\infty R_{zz}(\tau) d\tau, \quad (10)$$

where  $v_{g,z}(\tau)$  is the vertical velocity of a gas element (or a tracer particle) at time  $\tau$ , and  $R_{zz}(\tau) \equiv \langle v_{g,z}(\tau) v_{g,z}(0) \rangle$  is the auto correlation function for  $v_{g,z}$ . In a steady turbulence, we have  $R_{zz}(-\tau) = R_{zz}(\tau)$ . If we define the

power spectrum of turbulence as the Fourier transform for the auto correlation function

$$\hat{E}_{g,z}(\omega) = \frac{1}{2\pi} \int_{-\infty}^\infty R_{zz}(\tau) e^{i\omega\tau} d\tau, \quad (11)$$

we then have

$$R_{zz}(0) = \langle v_{g,z}^2 \rangle = \int_{-\infty}^\infty \hat{E}_{g,z}(\omega) d\omega. \quad (12)$$

If the ensemble average in  $R_{zz}(\tau)$  is defined as averaging  $v_{g,z}(t+\tau)v_{g,z}(t)$  of a fluid element over a long time, i.e.,  $\lim_{T \rightarrow +\infty} 1/T \times \int_{-T/2}^{T/2} v_{g,z}(t+\tau)v_{g,z}(t) dt$ , we have  $\hat{E}_{g,z}(\omega) = (2\pi/T) |\hat{v}_{g,z}(\omega)|^2$  by the convolution theorem.

One important quantity to characterize turbulence is the integral timescale or the eddy time, defined as  $t_{\text{eddy},z} \equiv \int_0^\infty R_{zz}(\tau)/R_{zz}(0) d\tau$ . If turbulence is isotropic, we have  $t_{\text{eddy},z} \sim t_{\text{eddy},x} \sim t_{\text{eddy},y}$  so that a single  $t_{\text{eddy}}$  is used to denote all three. If we set  $\omega = 0$  in Equation (11) and use Equation (10) and the definition of  $t_{\text{eddy}}$ , we have

$$\hat{E}_{g,z}(0) = \frac{1}{2\pi} \int_{-\infty}^\infty R_{zz}(\tau) d\tau = \frac{\langle v_{g,z}^2 \rangle t_{\text{eddy}}}{\pi} \quad (13)$$

and thus

$$D_{g,z} = \pi \hat{E}_{g,z}(0) = \langle v_{g,z}^2 \rangle t_{\text{eddy}}, \quad (14)$$

which suggests that the diffusion coefficient only depends on the power spectrum at  $\omega = 0$ , and  $D_{g,z}$  is the product of the mean squared velocity and the eddy time.

If we plug  $D_{g,z} = \langle v_{g,z}^2 \rangle t_{\text{eddy}}$  into Equations (7) and (8), the disk scale height for small particles ( $t_s \ll \Omega^{-1}$ ) in stratified disks is

$$h_d = \frac{h}{\sqrt{h^2 \Omega^2 t_s / (\langle v_{g,z}^2 \rangle t_{\text{eddy}}) + 1}}, \quad (15)$$

and the dust scale height in unstratified disks is

$$h_d = \sqrt{\frac{\langle v_{g,z}^2 \rangle t_{\text{eddy}}}{\Omega^2 t_s}}. \quad (16)$$

In order to derive the scale height for particles with any  $t_s$ , Youdin & Lithwick (2007) solve the Langevin equation (Equation (5)) using the Fourier transform, and derive

$$\hat{v}_{d,z} = \frac{\omega}{\omega + it_s(\Omega^2 - \omega^2)} \hat{v}_{g,z}. \quad (17)$$

The scale height is

$$\begin{aligned} h_d^2 &= \lim_{T \rightarrow +\infty} \frac{1}{T} \int_{-T/2}^{T/2} |z|^2 dt = \frac{2\pi}{T} \int_{-\infty}^\infty |\hat{z}_d|^2 d\omega \\ &= \int_{-\infty}^\infty |\hat{z}_d|^2 \frac{\hat{E}_g}{|\hat{v}_{g,z}|^2} d\omega, \end{aligned} \quad (18)$$

where  $\hat{z}_d = i\hat{v}_{d,z}/\omega$ .

Up to this point, the detailed form of the power spectrum of turbulence has not been assumed. In order to proceed, Youdin & Lithwick (2007) assume the turbulence power spectrum in a uniform unstratified disk is

$$\hat{E}_g(\omega) = \frac{\langle v_g^2 \rangle}{\pi} \frac{t_{\text{eddy}}}{1 + \omega^2 t_{\text{eddy}}^2}, \quad (19)$$

so that Equation (18) can be integrated to be

$$h_d^2 = \frac{D_{g,z}}{\Omega^2 t_s} \frac{1 + \text{St}}{1 + \text{St} + \text{St}(t_{\text{eddy}}\Omega)^2} \quad (20)$$

where the Stokes number ( $\text{St}$ ) is defined as  $\text{St} \equiv t_s/t_{\text{eddy}}$ . When  $\text{St} \ll 1$ , Equation (20) reduces to Equation (16). In the opposite limit, when  $\text{St} \gg 1$ , Equation (20) can also be approximated by Equation (16) (Carballido et al. 2006) as long as  $t_{\text{eddy}} \sim \Omega^{-1}$ .

Particle diffusion in the radial direction is defined in a similar way as that in the vertical direction (Equation (9))

$$D_{d,x} \equiv \frac{1}{2} \frac{d\langle x_d^2 \rangle}{dt}. \quad (21)$$

Then, the rest derivation of  $D_{d,x}$  is similar to the derivation above, except that it involves epicyclic oscillation, and both  $v_r$  and  $v_\phi$  need to be considered. Assuming the power spectra of  $v_{g,x}$ ,  $\delta v_{g,y}$ , and  $v_{g,z}$  have similar forms,<sup>4</sup> Youdin & Lithwick (2007) derive the particle radial diffusion coefficient as

$$D_{d,x} = t_{\text{eddy}} \frac{\langle v_{g,x}^2 \rangle + 4T_s^2 \langle \delta v_{g,y}^2 \rangle + 4T_s \langle v_{g,x} \delta v_{g,y} \rangle}{(1 + T_s^2)^2}. \quad (22)$$

For isotropic turbulence  $\langle v_{g,x}^2 \rangle = \langle \delta v_{g,y}^2 \rangle$ ,  $\langle v_{g,x} \delta v_{g,y} \rangle = 0$ , Equation (22) is reduced to

$$D_{d,x} = D_{g,x} \frac{1 + 4T_s^2}{(1 + T_s^2)^2}. \quad (23)$$

where  $D_{g,x} = \langle v_{g,x}^2 \rangle t_{\text{eddy}}$ .

Finally, if  $D_{g,x}$  (Equation (23)) is given, the evolution of the dust disk surface density (Equation (1)) is fully determined. If  $D_{g,z}$  is given, the vertical structure of the dust disk is also determined (Equation (20)). To get  $D_{g,x}$  and  $D_{g,z}$ , we need to know  $\langle v_{g,x}^2 \rangle$ ,  $\langle v_{g,z}^2 \rangle$  and  $t_{\text{eddy}}$ . Conveniently, previous ideal MHD simulations suggest that  $t_{\text{eddy}}$  is simply around  $\Omega^{-1}$  (Fromang & Papaloizou 2006; Carballido et al. 2011).

In the following, we will study the surface density evolution and vertical structure of the dust disk in our simulations, and compare the results with those derived from the simple one-dimensional (1-D) and analytical models (Equations (1) and (16)). Although the 1-D and analytical models that assume  $t_{\text{eddy}} \sim \Omega^{-1}$  can reproduce dust distribution in ideal MHD simulations very well,  $t_{\text{eddy}} \sim \Omega^{-1}$  breaks down in MHD simulations with AD (§4). Then we perform local shearing box simulations to directly measure  $t_{\text{eddy}}$  and the power spectrum of turbulence, which will be compared with the power spectrum normally assumed (Equation (19)) (§5).

### 3. SIMULATIONS

The gas dynamics is computed using Athena (Stone et al. 2008), a higher-order Godunov scheme for hydrodynamics and magnetohydrodynamics using the piecewise parabolic method for spatial reconstruction (Colella & Woodward 1984), the corner transport upwind method

for multidimensional integration (Colella 1990), and constrained transport to conserve the divergence-free property for magnetic fields (Gardiner & Stone 2005, 2008). Cylindrical grids (Skinner & Ostriker 2010) are used to simulate MRI turbulence in global disks.

We use the particle integrator in Athena (Bai & Stone 2010; Zhu et al. 2014) to study the motion of dust particles in MRI turbulent disks. Dust particles are implemented as Lagrangian particles with the dust-gas drag term, following

$$\frac{d\mathbf{v}_i}{dt} = \mathbf{f}_i - \frac{\mathbf{v}_i - \mathbf{v}_g}{t_s}, \quad (24)$$

where  $\mathbf{v}_i$  and  $\mathbf{v}_g$  are the velocity vectors for particle  $i$  and the gas,  $\mathbf{f}_i$  is the gravitational force from the central star, and  $t_s$  is the dust stopping time due to gas drag. This equation is integrated with time using second-order integrators which can preserve the geometric properties of particle orbits (Zhu et al. 2014). Since in a real disk, the vertical gravity is very small close to the disk midplane, the unstratified disk setup is a good approximation for simulating the gas dynamics there (e.g. within half the gas scale height of the disk). However, even close to the disk midplane, dust can have a significant stratification since it can have a much smaller scale height than the gas due to dust settling. Thus, we include the vertical gravitational force for dust particles, in which case dust is allowed to settle toward the disk midplane.

#### 3.1. The Gas Disk

The gas disk is unstratified so that the gas density is constant in the  $z$  direction. The initial radial profile of the gas disk is

$$\rho_g(r, \phi, z) = \rho_{g,0} \left( \frac{r}{r_0} \right)^{-1} \quad (25)$$

$$T(r, \phi, z) = T_0 \left( \frac{r}{r_0} \right)^{-1/2}, \quad (26)$$

so that  $h/r = c_s/v_\phi = (h/r)_{r=r_0} (r/r_0)^{1/4}$ . We choose  $\rho_{g,0} = 1$ ,  $r_0 = 1$ , and  $(h/r)_{r=1} = 0.1$ . The disk is locally isothermal at each  $r$ , which means that it has the same temperature in the  $\phi - z$  plane at a given  $r$ . The local isothermal assumption is valid when the viscous/turbulent heating is much less efficient than the stellar irradiation so that the disk temperature is only controlled by the stellar irradiation (Kenyon & Hartmann 1987; Calvet et al. 1991). Since we are interested in the outer disk beyond 10s of AU, where ambipolar diffusion dominates, the local isothermal assumption is a good approximation. To compare with the 1-D dust surface density evolution equation, we use  $\rho_g$  in the unstratified disk to represent  $\Sigma_g$ , and choose the slope of  $\rho_g$  as -1 (Equation 25), which is the slope of the surface density in the  $\alpha$  disk similarity solution. However, in a realistic stratified disk, the midplane density has a steeper gradient along the radius than the surface density (e.g. demonstrated in Appendix B of Zhu et al. 2014). Thus, particles drift significantly slower in our simulations than in a realistic disk. Such slower drift allows us to evolve the dust disk for a longer time but also allow more particles concentrate in pressure bumps (§6.2).

<sup>4</sup> Youdin & Lithwick (2007) use the local shearing box approximation so that  $x$  is the  $r$  direction, and  $y$  is the  $\phi$  direction.

TABLE 1  
 GLOBAL SIMULATIONS

Run Name	Resolution	B	t $2\pi/\Omega(r_0)$	Par. No. Millions	$\langle v_{g,r}^2 \rangle$ $(c_s^2)$	$\langle \delta v_{g,\phi}^2 \rangle$ $(c_s^2)$	$\langle v_{g,z}^2 \rangle$ $(c_s^2)$	$\langle \rho v_{g,r} \delta v_{g,\phi} \rangle / \langle \rho \rangle$ $(c_s^2)$	$\langle -B_r B_\phi \rangle / \langle \rho \rangle$ $(c_s^2)$	$\alpha^a$
V1e4	$576 \times 1024 \times 32$	Vert. $\beta_0 = 1e4$	100	1	0.036	0.015	0.010	$7.5e-3$	0.028	0.035
V1e5	$576 \times 1024 \times 32$	Vert. $\beta_0 = 1e5$	100	1	0.031	0.010	$6.8e-3$	$5.3e-3$	0.017	0.022
T1e2	$576 \times 1024 \times 32$	Tor. $\beta_0 = 100$	100	1	0.031	0.015	0.010	$7.5e-3$	0.024	0.032
T1e3	$576 \times 1024 \times 32$	Tor. $\beta_0 = 1e3$	100	1	0.027	0.012	$7.5e-3$	$6.2e-3$	0.018	0.025
AD1e3	$576 \times 1024 \times 32$	Vert. $\beta_0 = 1e3$	100	1	$5.2e-3$	$1.6e-3$	$1.2e-3$	$8.8e-4$	$1.5e-3$	$2.4e-3$
AD2.5e4	$576 \times 1024 \times 32$	Vert. $\beta_0 = 2.5e4$	100	1	$3.9e-3$	$1.9e-4$	$3.4e-5$	$5.0e-4$	$1.8e-4$	$6.4e-4$
High reso. (32/h)										
V1e4H	$1152 \times 2048 \times 64$	Vert. $\beta_0 = 1e4$	60	300	0.032	0.021	0.013	$7.7e-3$	0.030	0.039
T1e2H	$1152 \times 2048 \times 64$	Tor. $\beta_0 = 100$	100	1	0.037	0.018	0.011	0.010	0.023	0.033
AD1e3H	$1152 \times 2048 \times 64$	Vert. $\beta_0 = 1e3$	100	1	$7.0e-3$	$2.0e-3$	$1.7e-3$	$9.8e-4$	$1.7e-3$	$2.8e-3$

$$^a \alpha \equiv \frac{\langle -B_r B_\phi \rangle}{\langle \rho \rangle c_s^2} + \frac{\langle \rho v_{g,r} \delta v_{g,\phi} \rangle}{\langle \rho \rangle c_s^2}.$$

Turbulence is driven by the MRI in both ideal and non-ideal MHD simulations. Following Bai & Stone (2011), we include the effect of AD by modifying the induction equation as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} - \frac{4\pi}{c} \eta_A \mathbf{J}_\perp \right), \quad (27)$$

where  $\mathbf{B}$  is the magnetic field,  $\mathbf{J}_\perp = (\nabla \times \mathbf{B})_\perp$  is the component of the current density that is perpendicular to the direction of the magnetic field, and  $\eta_A$  is the ambipolar diffusivity  $\eta_A \equiv v_A^2 / (\gamma \rho_i)$  in which  $v_A$  is the Alfvén speed and  $\gamma \rho_i$  is the neutral-ion collision frequency. The effect of AD in disks can be characterized by the dimensionless parameter  $\text{Am}$  (Chiang & Murray-Clay 2007),

$$\text{Am} \equiv \frac{\gamma \rho_i}{\Omega}, \quad (28)$$

which is the number of collisions for a neutral molecule with ions in the dynamical timescale  $1/\Omega$ . We adopt  $\text{Am} = 1$  which is the typical value for the protoplanetary disk from 10 to 100 AU (Bai 2011a, 2014). Thus, the ambipolar diffusivity  $\eta_A$  becomes  $\eta_A = v_A^2 / \Omega$ .

For ideal MHD simulations, the disk is threaded by either net vertical or net toroidal magnetic fields. For AD simulations, we only study disks threaded by net vertical fields since net toroidal fields generate little turbulence in disks dominated by AD (Bai & Stone 2011). The initial magnetic fields have a constant plasma  $\beta = 8\pi \rho c_s^2 / B^2$  everywhere in the whole disk. The initial configuration of magnetic fields is shown in Table 1. Ideal MHD runs with net vertical and toroidal fields have names starting with “V” and “T” respectively, while AD runs start with “AD”. For each magnetic field geometry, we have varied the initial plasma  $\beta$  by at least one order of magnitude.

Our cylindrical grids span from 0.5 to 4 in the  $r$  direction, 0 to  $2\pi$  in the  $\phi$  direction, and -0.1 to 0.1 in the  $z$  direction. The vertical domain of  $z = [-0.1, 0.1]$  is equivalent to  $2h$  at  $r = 1$ ,  $4.8h$  at the inner boundary ( $r = 0.5$ ) and  $0.7h$  at the outer boundary ( $r = 4$ ). The grid is uniformly spaced in all  $r$ ,  $\phi$  and  $z$  directions. Our standard simulations have the resolution of  $576 \times 1024 \times 32$  in the  $r$ ,  $\phi$ , and  $z$  directions, which is 16 grids per  $h$  at  $r = 1$  in all three directions. The high resolution runs with the resolution of  $1152 \times 2048 \times 64$  have 32 grids per  $h$  at  $r = 1$ .

At the outer radial boundary, the physical quantities in ghost zones are set to be fixed at the initial values

(as in Zhu et al. 2014). Such a boundary can absorb waves quite efficiently in numerical codes using Godunov-type schemes. At the inner boundary, the open boundary condition used in Sorathia et al. (2012) has been applied to allow mass accretion. Periodic boundary conditions have been applied in both  $\phi$  and  $z$  directions.

### 3.2. Particle Component

In the initial condition, we distribute dust particles in a way that leads to the same radial profile of surface density as the gas (Equation (25)).<sup>5</sup> In detail, to ensure that the surface density of the dust has a slope of -1, each particle is placed in the disk at a constant probability in both  $r$  and  $\phi$  directions. All particles are initially placed at the disk midplane with circular Keplerian speeds.

We evolve seven types/species of particles simultaneously with the gas. For each particle type, there are  $10^6$  particles. We assume that all these particles are in the Epstein regime, so that the dust stopping time (Whipple 1972; Weidenschilling 1977, we use the notation from Takeuchi & Lin 2002) is

$$t_s = \frac{s \rho_p}{\rho_g v_T}, \quad (29)$$

where  $\rho_g$  is the gas density,  $s$  is the dust particle radius,  $\rho_p$  is the dust particle density (we choose  $\rho_p = 1 \text{ g cm}^{-3}$ ),  $v_T = \sqrt{8/\pi} c_s$ , and  $c_s$  is the gas sound speed.

To make our results general, we do not specify the length and mass unit in our simulations. Each particle type in our simulations has one certain size ( $s$ ) in Equation (29). With our chosen particle size ( $s$ ), the dust stopping time for each particle type at  $r = 1$  in the initial condition is shown in Table 2. Note that  $t_s$  will evolve with time in the simulation since  $\rho_g$  and  $v_T$  are changing with time along the particle’s trajectory.

Given a realistic protoplanetary disk structure, particles in our simulations can be translated to real particles having some physical sizes in protoplanetary disks. Using Equation (29) and  $c_s = h\Omega$ , a dust particle with size  $s$  at the midplane of a realistic disk ( $\rho_{\text{mid}} = \Sigma / (\sqrt{2\pi} h)$ )

<sup>5</sup> For passive particles, we can arbitrarily scale the dust surface density without affecting the dynamics. To compare with observations, we only need to scale the dust density using the realistic dust-to-gas mass ratio.

has

$$T_s = t_s \Omega = \frac{\pi s \rho_p}{2 \Sigma_g} = 1.57 \times 10^{-3} \frac{\rho_p}{1 \text{ g cm}^{-3}} \frac{s}{1 \text{ mm}} \frac{100 \text{ g cm}^{-2}}{\Sigma_g}. \quad (30)$$

For example, if we are studying an  $\alpha$  disk similarity solution with  $\alpha = 0.01$ ,  $\dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ , and  $T = 221(R/\text{AU})^{-1/2} \text{ K}$ , the disk surface density is then  $\Sigma_g = 178(r/\text{AU})^{-1} \text{ g cm}^{-2}$ . Using Equation (30), we can derive that the smallest particle type in our simulations with  $T_s = 0.007041$  at  $r = 1$  corresponds to 1.5 mm particles at 5 AU and 0.4 mm particles at 20 AU in such a disk. With different realistic disk structures, Table 2 gives the real particle sizes that our simulated particle types correspond to. To relate our results with the latest *ALMA* observation on Oph IRS 48 (van der Marel et al. 2013, Bruderer et al. 2014), we give the corresponding size of particles in Oph IRS 48 in the last column. However, with a realistic surface density, some particle types are in the Stokes rather than the Epstein regime. Those particle types can only be considered as a numerical experiment to explore the effect of a large stopping time on the particle distribution, rather than a realistic model of such particles.

TABLE 2  
CORRESPONDING PARTICLE SIZES

Par	$t_s \Omega$ At $r=1$	Par. Size At 5 AU in a MMSN Disk <sup>a</sup>	Par. Size At 5 AU in $\alpha$ Disk <sup>b</sup>	Par. Size at 20 AU in $\alpha$ Disk	Par. Size At 30 AU in Oph IRS 48 <sup>c</sup>
a	0.007041	6.8 mm	1.5 mm	0.4 mm	3 $\mu\text{m}$
b	0.07041	6.8 cm	15 mm	4 mm	30 $\mu\text{m}$
c	0.7041	Stokes	15 cm	4 cm	0.3 mm
d	7.041	Stokes <sup>d</sup>	Stokes	40 cm	3 mm
e	70.41	Stokes	Stokes	4 m	3 cm
f	704.1	Stokes	Stokes	Stokes	30 cm
g	7041	Stokes	Stokes	Stokes	3 m

<sup>a</sup> A MMSN disk has  $\Sigma_g = 1700(r/\text{AU})^{-1.5} \text{ g cm}^{-2}$ .

<sup>b</sup> The  $\alpha$  disk has  $\Sigma_g = 178(r/\text{AU})^{-1} \text{ g cm}^{-2}$ , which is the surface density of a constant  $\alpha = 0.01$  accretion disk with  $\dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ , and  $T = 221(r/\text{AU})^{-1/2} \text{ K}$ .

<sup>c</sup> Oph IRS 48 disk with a  $2 \text{ M}_\odot$  central star,  $\Sigma_g = 1.92(r/\text{AU})^{-1} \text{ g cm}^{-2}$  and  $T = 542(r/\text{AU})^{-1/2} \text{ K}$  (Bruderer et al. 2014).

<sup>d</sup> Particles in the Stokes regime, see text.

The turbulent disk with both gas and dust is evolved for 100 orbits. Throughout the paper, an orbit is defined as the orbital period at  $r = 1$  ( $2\pi/\Omega(r = 1)$ ). During this time, most of our particle types have reached vertical equilibrium since the settling timescale of dust particles ( $\Omega^{-1}(T_s + T_s^{-1})$ ) is only 22 orbits for the smallest particles in our simulations (Par. a).

#### 4. RESULTS

MRI turbulence is significantly suppressed when AD dominates in the disk, as shown in Table 1. In ideal MHD runs, both V1e4 and T1e2 have similar total stresses which are equivalent to  $\alpha \sim 0.03$ , and V1e5 and T1e3 have  $\alpha \sim 0.02$ . When AD dominates, even strong vertical fields ( $\beta_0 = 1000$  in AD1e3) can only lead to  $\alpha \sim 0.002$ . This strong suppression of turbulence by AD is consistent with previous studies (Bai & Stone 2011). Since the density, temperature, and magnetic fields vary radially in our setup,  $\alpha$  varies radially too. By measuring the

radial profile of  $\alpha$  in our simulations, we find  $\alpha \propto r^{-5/4}$ . This trend is consistent with the  $\alpha$  predictor found in unstratified shearing box simulations (Hawley et al. 1995) that  $\alpha \propto L_z \Omega^2 \lambda_c / c_s^2$  where  $L_z$  is the size of the box in the  $z$  direction, and  $\lambda_c \sim 2\pi v_A / \Omega \sim \sqrt{\pi/\rho} B / \Omega$ . With a constant  $\beta_0$  throughout the disk, we have  $B \propto r^{-3/4}$ ,  $\lambda_c \propto r^{5/4}$ , and the predictor above suggests that  $\alpha$  is indeed  $\propto r^{-5/4}$ .

Since the mean squared velocities (i.e.,  $\langle v_{g,r}^2 \rangle$ ,  $\langle v_{g,\phi}^2 \rangle$ ,  $\langle v_{g,z}^2 \rangle$ ) together with the eddy time ( $t_{\text{eddy}}$ ) determine the dust diffusion coefficients (e.g.  $D_{d,x}$ ,  $D_{d,z}$ ) and the dust scale height (Equations (14), (20), (22), (23)), we measure the averaged mean squared velocities in all our simulations, as also shown in Table 1. All our runs have  $\langle v_{g,r}^2 \rangle \gtrsim \langle \delta v_{g,\phi}^2 \rangle \gtrsim \langle v_{g,z}^2 \rangle$ . The ratio between  $\langle v_{g,r}^2 \rangle$  and  $\langle v_{g,z}^2 \rangle$  is normally 3 in ideal MHD runs, while this ratio can be significantly larger in AD runs. Another trend is that  $\langle v_{g,r}^2 \rangle \sim \alpha c_s^2$  in ideal MHD runs, while  $\langle v_{g,r}^2 \rangle > \alpha c_s^2$  in AD runs. Nevertheless, since  $\langle v_{g,r}^2 \rangle$  is several times of  $\langle v_{g,z}^2 \rangle$  in all our runs, we expect that MRI turbulence should lead to larger radial diffusion coefficients than vertical diffusion coefficients. However, as shown in §4.2 and §5, this expectation is not true for turbulence generated by the MRI with AD due to the large eddy time for  $v_z$  in these disks.

During 100 orbits, the dust distribution has evolved significantly due to both radial drift and turbulent diffusion. Figure 1 shows the disk surface density for the gas and different types of particles at 60 orbits in V1e4H. In particle panels, the ratio of the dust to the gas surface density is shown. The smallest particles (Par. a) couple with the gas so well that the density ratio is almost 1 throughout the disk. Bigger particles (e.g., Par. b and c) which drift faster in disks have smaller radial extents than the gas. They are also more concentrated in spiral arms, suggesting that particles drift azimuthally responding to the gas turbulent structures. Particles with  $T_s > 1$  (e.g., Par. d and e) start to decouple from the gas. Due to their long coupling times with the gas, they cannot respond to density fluctuations occurring faster than the orbital timescale. Thus, the inhomogeneity of the gas fluctuations in the azimuthal direction (e.g. spiral arms) only has an average effect on them, causing more axisymmetric particle distributions. In the following, we will study particle radial drift, and vertical settling in more detail.

##### 4.1. Evolution of Dust Surface Density

In order to test if the evolution of dust in our 3-D turbulent simulations can be reproduced by a simple 1-D dust model (Equation (1)), we solve Equations (1) to (3) with the same disk structure in 3-D simulations. Equations (1) to (3) are solved using the operator split finite difference scheme similar to Stone & Norman (1992). The diffusion term in Equation (2) is added using the central difference scheme, while the drift term in Equation (3) is added using the van Leer upwind method. At the radial boundaries, the dust density is set to be zero. In order to solve the evolution of dust, we also need to specify how gas ( $\Sigma_g(t)$ ) evolves with time in Equations (2) and (3). To approximate the evolution of the gas disk,

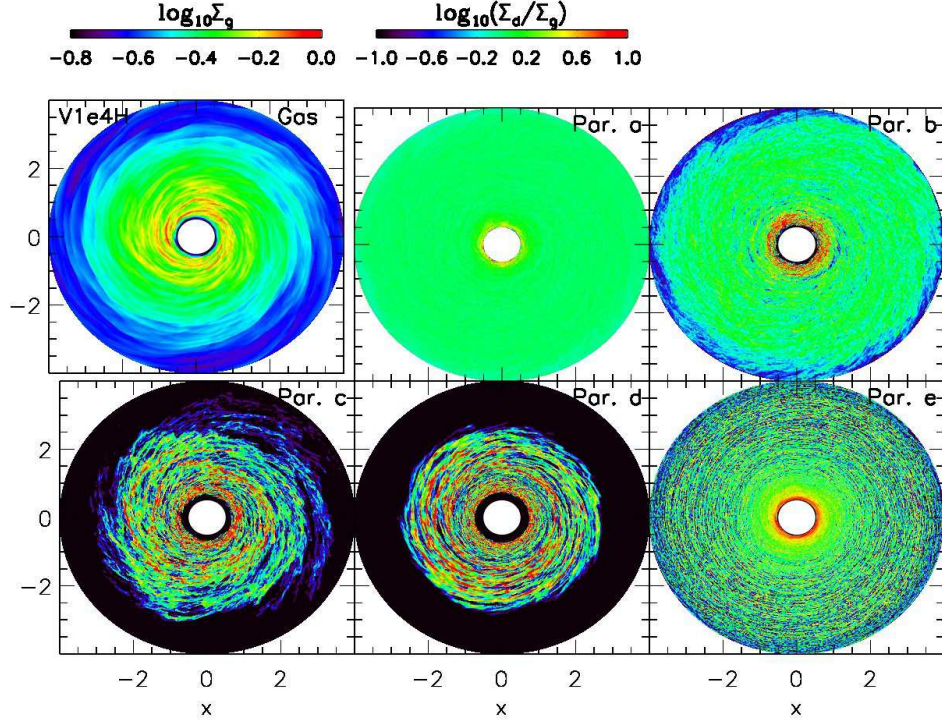


FIG. 1.— Disk surface density at 60 orbits for the ideal MHD simulation V1e4H. The upper left panel shows the surface density of the gas disk, while the other five panels show the ratio between the dust and gas surface density. The smallest particles (Par. a) couple with the gas so well that the density ratio is close to 1. With increasing particle size from Par. a to Par. c, the disk becomes smaller due to the faster radial drift, and dust concentrates more within the spiral arms. Par. d and e, with  $T_s > 1$ , start to decouple from the gas and the concentration is more axisymmetric.

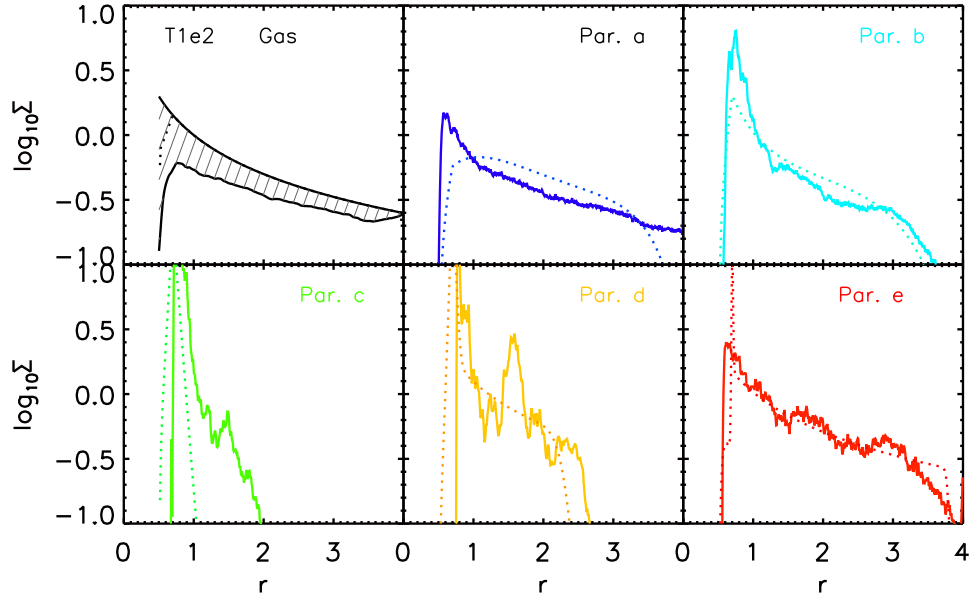


FIG. 2.— Azimuthally averaged disk surface density for the gas (black curves) and different types of particles (colored curves) in run T1e2 at 100 orbits. In the gas panel, the upper black solid curve is the initial gas surface density, while the lower black solid curve is the gas surface density at 100 orbits. The shaded region is thus due to disk accretion. The dotted curves are from the 1-D dust model (Equation (1)) with the fixed  $\Sigma_g$  (Equation (31)) and  $D_{d,r}$ . This 1-D model generally agrees with simulation results, but it leads to a too small disk for Par. c, and unable to reproduce large-scale density peaks for Par. d and Par. e.

two 1-D gas models have been constructed using MHD simulations with different degrees of sophistication. The



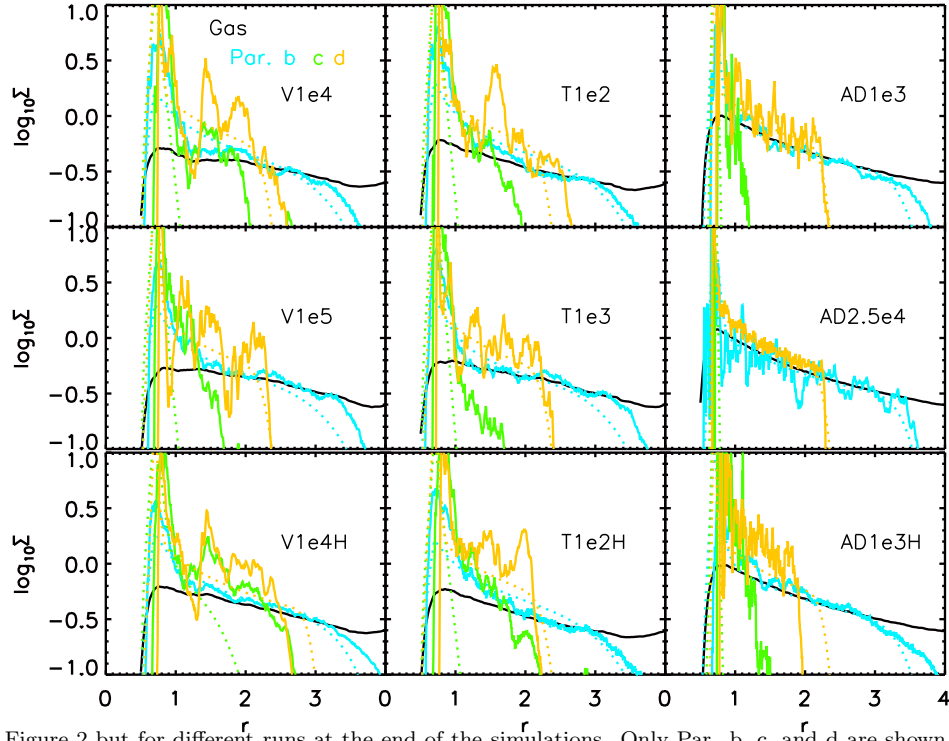


FIG. 3.— Similar to Figure 2 but for different runs at the end of the simulations. Only Par. b, c, and d are shown. In each panel, the dotted curves are from the 1-D model used in Figure 2.

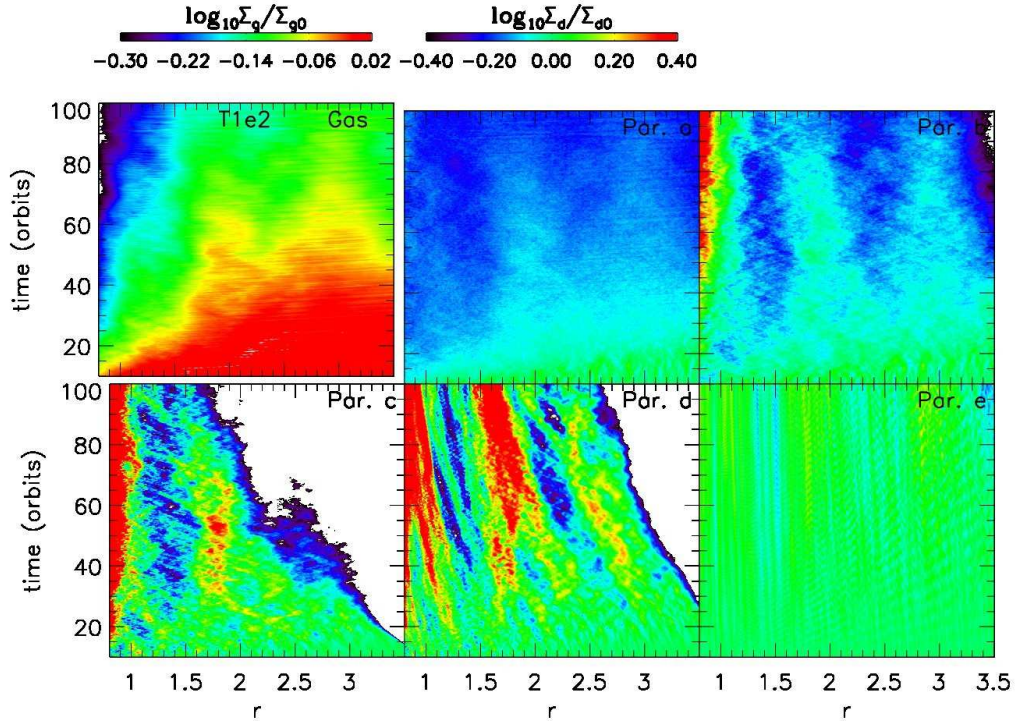


FIG. 4.— Space time diagram of the relative surface density in the radial direction for gas and various types of particles in run T1e2. In the gas panel, besides the decrease of the surface density due to gas accretion, two density peaks are visible at  $r \sim 2$  and  $r \sim 3$ . Since these gas features can trap dust particles, the density peaks become more apparent in the Par. b panel, most prominent in the Par. c and d panels, and also noticeable in the Par. e panel.

first model does not require knowing how the gas surface density fluctuates with time due to MRI turbulence. A fixed gas surface density has been applied. In the sec-

ond model, the gas surface density at every time step is directly inputted from MHD simulations at that time.

In the first 1-D model, based on the fact that the gas



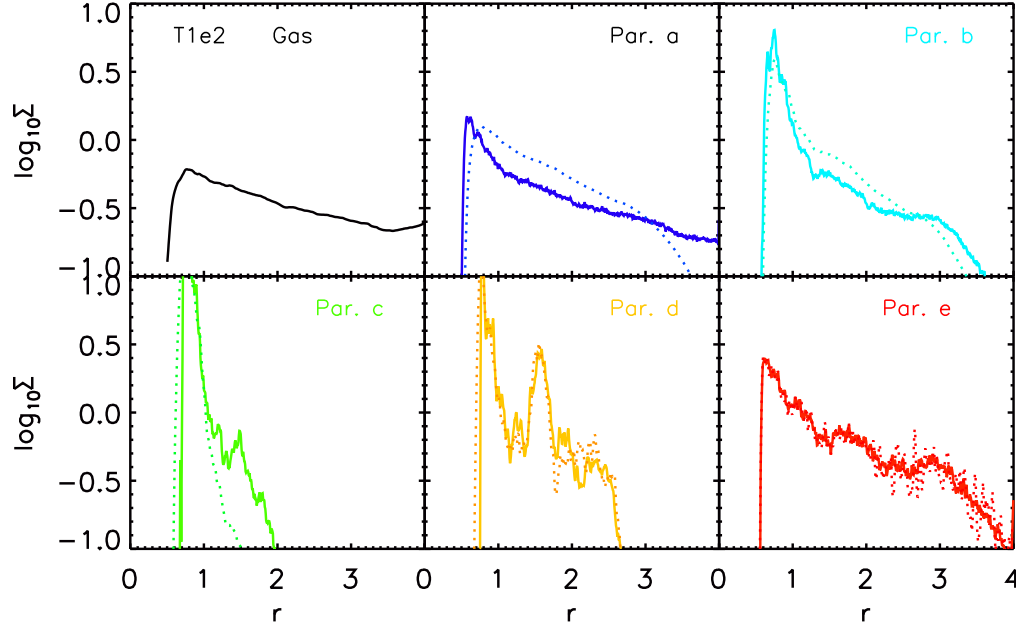


FIG. 5.— Similar to Figure 2, but the dotted curves are from the refined 1-D model with  $\Sigma_g$  and  $D_{d,r} = \alpha c_s^2 / \Omega$  inputted from the MHD simulation. The agreement between this refined 1-D model and the simulation is significantly improved compared with Figure 2. In particular, particle trapping in the density peaks is very well reproduced (Par. d and e), suggesting the refined 1-D model is adequate to study particle trapping in zonal flows. However, even this 1-D model could not fully reproduce the distribution of Par. c, which has  $T_s \sim 1$ .

disk does not evolve much within 100 orbits, we fix the gas surface density as

$$\begin{aligned} \Sigma_g &= \Sigma_0 r^{-1} & \text{at } r \geq 0.7 \\ \Sigma_g &= \Sigma_0 r^{-1} \exp((r - 0.7)/0.2) & \text{at } 0.5 \leq r \leq 0.7 \end{aligned} \quad (31)$$

where  $\Sigma_0 = 1$ . The initial dust surface density equals the gas surface density. The exponential cut-off within  $r = 0.7$  is to mimic the gas surface density in MRI global simulations where the inner disk within  $r = 0.7$  is depleted due to the inner open boundary condition.  $D_{d,r}$  is equal to  $D_{d,x}$  in Equation (23). Since  $t_{\text{eddy}} = \Omega^{-1}$  is a good assumption in ideal MHD (§5 and Carballido et al. 2011),  $D_{g,x}$  is simplified to  $\langle v_{g,r}^2 \rangle \Omega^{-1}$ . Furthermore, with  $\langle v_{g,r}^2 \rangle \sim \alpha c_s^2$  and  $\alpha \propto r^{-5/4}$  for our disk setup, we have  $D_{g,x} \propto r^{-1/4}$ . Thus, we set  $D_{g,x} = \langle v_{g,r}^2 \rangle_{r=1} \Omega_0^{-1} (r/r_0)^{-1/4}$ , where  $\langle v_{g,r}^2 \rangle_{r=1}$  is given in Table 1.

The comparison between this simple 1-D model and the 3-D turbulent simulation T1e2 at 100 orbits is shown in Figure 2, which demonstrates that the main features in the 3-D simulation can be reproduced by the 1-D model. The solid curves in Figure 2 are the azimuthally averaged surface density for the gas (black curve) and different particle types (colored curves) in the simulation, while the dotted curves are the same quantities from the simple 1-D model. In the gas panel, the upper black solid curve is the initial gas surface density in the simulation, while the lower black solid curve is the gas surface density at 100 orbits. The shaded region is thus due to disk accretion, which confirms that the gas disk does not evolve much during 100 orbits since this is significantly

shorter than the viscous timescale. For most particle types, both the amplitude and size of the dust disks are similar between the 1-D model and 3-D simulations<sup>6</sup>

However, for Par. c and d having  $T_s \sim 1$ , there is a noticeable discrepancy between the simulation and the 1-D model. The 1-D model predicts that all Par. c should have drifted to the inner boundary, while the simulation has a significantly extended disk. For Par. d, not only the position of the outer edge is different but also dust in the MHD simulations show large amplitude density peaks. These peaks for Par. d are also present in other ideal MHD simulations (left two panels in Figure 3) where the disk is very turbulent.

These dust density peaks in simulations are related to the large-scale gas features induced by MRI turbulence (e.g., zonal flows). Examining the gas surface density closely in Figure 3 (black solid curves), we can see there are small amplitude density fluctuations in the gas disk. Although these fluctuations only change the gas surface density slightly, they can affect dust significantly since the dust drift speed sensitively depends on the slope of the gas surface density. The effect is that dust particles always drift to pressure peaks in disks. Figure 4 shows the space time diagram for the disk surface density in run T1e2. In order to illustrate the density fluctuations instead of the background density gradient, we have divided the surface density at  $t$  by the initial surface density. The top left panel shows the gas surface density. Besides the general trend of the decreasing gas surface

<sup>6</sup> The comparison is not good for Par. a at the outer disk edge since the boundary conditions are different in 3-D simulations and 1-D models.

density due to accretion, we can see two density peaks at  $r \sim 2$  and  $3$  which persist over the whole simulation. Since these gas features can trap dust particles, the density peaks become more apparent in the Par. b panel, most prominent in the Par. d panel, and also noticeable in the Par. e panel. More discussions on the amplitude of zonal flows in various simulations and how they trap particles are presented in §6.2.

To take into account the gas fluctuations induced by MRI turbulence, we construct a second 1-D evolutionary model for the dust disk. In this model, we first output the azimuthally averaged gas surface density ( $\Sigma_g$ ) and equivalent  $\alpha$  from MHD simulations at every 0.1 orbit. Then we solve Equations (1) to (3) by inputting this time evolving  $\Sigma_g$  and  $D_{d,r} \sim \alpha c_s^2 / \Omega$ . The dust will then respond to the evolving gas disk. Using this model for T1e2 run, we derive the surface density for dust at 100 orbits, which is plotted as the dotted curves in Figure 5. For Par. c, this model still produces a much smaller disk than the simulation, suggesting that 1-D axisymmetrically averaged disk models cannot reproduce the evolution of dust with  $T_s \sim 1$ , and non-axisymmetric density features in disks (e.g., spiral arms) affect the evolution of these particles significantly. However, this refined 1-D model reproduces the surface density evolution for other particle types very well. Especially for Par. d and e, the 1-D model reproduces the dust density peaks very accurately, confirming that the density peaks in the dust are due to axisymmetric structures in the gas disk (e.g., zonal flows).

In AD runs, since the turbulence is very weak, the evolution of the dust in our simulations is dominated by the radial drift. To study radial turbulent diffusion in AD cases, we need to carry out shearing box simulations where the dust radial drift is zero. These simulations are carried out and studied in §5.

#### 4.2. Dust Vertical Settling

Besides the radial drift, particles will also settle to the disk midplane. In turbulent disks, a steady density profile in the  $z$  direction is established when particle settling is balanced by turbulent diffusion, and the profile can be derived analytically as given by Equations (15) and (16).

Thus, we compare the dust vertical structure in our simulations with these analytical models in Figure 6. The solid curves are the vertical dust density profiles in our simulations at  $r = 1$ , while the dotted curves have Gaussian density profiles with the scale height ( $h_d$ ) from Equation (16). Although the simulations cannot resolve dust scale height of particles with  $T_s > 1$  (e.g. Par. d and e), we find Par. a to c in ideal MHD simulations can be well fitted by the analytical model assuming  $t_{\text{eddy}} = (2\Omega)^{-1}$ . This is consistent with previous MHD simulations that  $h_d$  with  $t_{\text{eddy}} \sim \Omega^{-1}$  can reproduce the dust vertical structure in ideal MHD simulations.

However, when AD dominates in disks, this analytical profile with  $t_{\text{eddy}} = (2\Omega)^{-1}$  significantly underestimates the dust scale height in simulations by more than a factor of 2 (right panels in Figure 6). To explain these relatively thick dust disks, Equation (16) requires that  $t_{\text{eddy}}$  has to be several times of  $\Omega^{-1}$  in AD runs which is much larger than  $t_{\text{eddy}} \sim \Omega^{-1}$  in ideal MHD runs.

### 5. SHEARING BOX SIMULATIONS

It is surprising that the presence of AD significantly affects  $t_{\text{eddy}}$  and  $D_{g,z}$ . To verify this result and unveil the physics behind it, we have carried out both unstratified and stratified shearing box simulations including Lagrangian dust particles.

#### 5.1. Unstratified Simulations

The unstratified simulations have a similar setup as Bai & Stone (2011). For most runs, the box size is  $[-0.5h, 0.5h] \times [-2h, 2h] \times [-h, h]$  in the  $x$ ,  $y$ , and  $z$  directions. For ideal MHD simulations, disks are threaded by net vertical, net toroidal, or zero net fields. For simulations with AD, Am is again chosen as one and only net vertical fields have been applied. The detailed simulation parameters are shown in Table 3. The nomenclature for run names in Table 3 is as following. In ideal MHD runs, “V”, “T”, and “Z” denote that the disk is threaded by net vertical, toroidal, or zero magnetic flux. In AD runs, “AD1”, “AD2”, and “AD3” represent the disk with initial plasma  $\beta$  of  $10^3$ ,  $10^4$ , and  $2.5 \times 10^4$ . “R32”, “R64”, and “R128” indicate that the numerical resolution is  $32/h$ ,  $64/h$ , and  $128/h$ .

In these simulations, there are seven types of particles with stopping times of  $T_s = \{10^{-3}, 10^{-2}, 10^{-1}, 1, 10, 10^2, 10^3\}$ , which are denoted as Par. A to G (different from Par. a to g in global simulations). For each type of particles, we have put  $10^4$  particles uniformly in the box. We have run these simulations to 100 orbits. Although the gas is unstratified, particles feel the gravitational force to the disk midplane similar to our global unstratified simulations.

Turbulence in these simulations has similar properties as in global unstratified simulations in §4. The averaged mean squared velocities, stresses, and equivalent  $\alpha$  are given in Table 3. The average is taken from the whole simulation box and over the time from snapshots at each orbit from 60 to 100 orbits. Similar to the global runs, turbulence is significantly suppressed by AD, and all the ideal MHD simulations have  $\alpha c_s^2 \sim \langle v_{g,x}^2 \rangle > \langle v_{g,y}^2 \rangle > \langle v_{g,z}^2 \rangle$ , while disks dominated by AD have  $\langle v_{g,x}^2 \rangle \gtrsim \langle v_{g,z}^2 \rangle$  and  $\langle v_{g,x}^2 \rangle > \alpha c_s^2$ .

These simulations have also confirmed that  $h_d$  with  $t_{\text{eddy}} \sim \Omega^{-1}$  significantly underestimates the dust scale height in disks dominated by AD. Figure 7 shows the dust vertical density profiles after being averaged at every orbit from 60 to 100 orbits. The profiles confirm the results found in our global simulations (e.g., Figure 6) that  $h_d$  with  $t_{\text{eddy},z} = (2\Omega)^{-1}$  (Equation (16)) can fit the dust density profiles in ideal MHD runs well (at least for Par. A to C with a resolved scale height), but a larger  $h_d$ —thus a larger  $D_{g,z}$  and  $t_{\text{eddy},z}$ —is needed to explain density profiles in simulations dominated by AD.

We have traced the trajectory of each individual particle in shearing box simulations, so that we can measure the dust diffusion coefficients directly using Equations (9) and (21). For each type of particles, we randomly pick 160 particles and measure their rates of change of  $x^2$  and  $z^2$  from 30 to 50 orbits. Then we average these rates to derive the diffusion coefficients in both radial and vertical directions. Our smallest particles couple with the gas so well that we use their  $D_d$  to represent the gas diffusion coefficients  $D_g$ . Both  $D_{g,x}$  and  $D_{g,z}$  are given in Table 3 for all our runs.  $t_{\text{eddy},x}$  and  $t_{\text{eddy},z}$  calculated

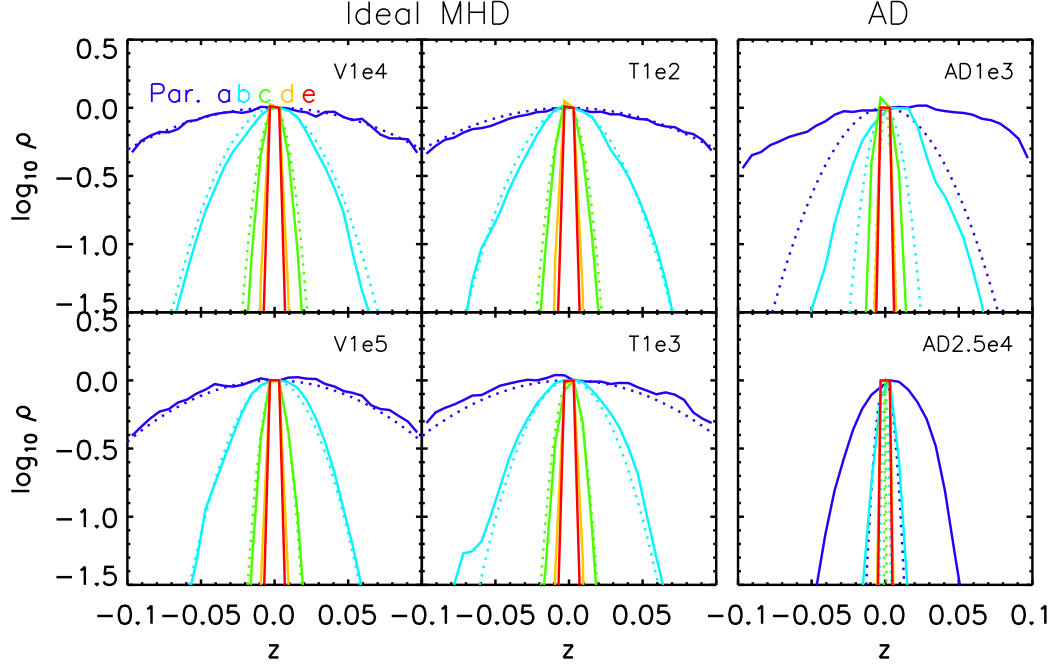


FIG. 6.— Azimuthally averaged density at  $r = 1$  with respect to disk height for different particle types (colored curves) in ideal and non-ideal MHD runs at 100 orbits. The dotted curves are the Gaussian profiles with  $h_d$  (Equation (16)) derived with  $t_{\text{eddy}} = (2\Omega)^{-1}$ . The agreement is very good for ideal MHD runs. For AD runs (the right panels), this  $h_d$  with  $t_{\text{eddy}} = (2\Omega)^{-1}$  significantly underestimates the dust scale height in simulations.

by  $D_{g,x}/\langle v_x^2 \rangle$  and  $D_{g,z}/\langle v_z^2 \rangle$  are also given in Table 3. All the ideal MHD runs have  $D_{g,x} \sim \langle v_x^2 \rangle \Omega^{-1}$  and  $D_{g,z} \sim \langle v_z^2 \rangle \Omega^{-1}$ , suggesting  $t_{\text{eddy}} \sim \Omega^{-1}$  for both  $v_x$  and  $v_z$ . This is again consistent with previous simulations (Johansen & Klahr 2005).

However, in all our AD runs, the measured  $D_{g,z}$  is three to four times of  $\langle v_z^2 \rangle \Omega^{-1}$ , so that  $t_{\text{eddy},z} \sim 3 - 4\Omega^{-1}$  which is significantly longer than the eddy time in ideal MHD ( $t_{\text{eddy},z} \sim \Omega^{-1}$ ). It is this large  $t_{\text{eddy},z}$  that causes the relatively large dust scale height in both global and local AD simulations. Another anomaly in AD runs is that  $t_{\text{eddy},x} \neq t_{\text{eddy},z}$ .  $t_{\text{eddy},x}$  can be 5 times larger than  $\Omega^{-1}$  (AD1 with  $\beta_0 = 10^3$ ), comparable to  $\Omega^{-1}$  (AD2 with  $\beta_0 = 10^4$ ), or even 4 times smaller than  $\Omega^{-1}$  (AD3 with  $\beta_0 = 2.5 \times 10^4$ ).

If we plug the measured  $D_{g,z}$  or  $t_{\text{eddy},z}$  from Table 3 into Equation (20) to derive the analytical density profiles, the resulting Gaussian profiles can fit the simulation results in both ideal MHD and AD runs very well. These density profiles using  $D_{g,z}$  or  $t_{z,\text{eddy}}$  in Table 3 are shown in Figure 8 as the dashed curves, compared with the simulation results as the solid curves. This confirms that the large  $D_{g,z}$  and  $t_{z,\text{eddy}}$  are responsible for the thick dust disks in AD runs.

The anomaly of  $t_{\text{eddy}}$  in AD runs indicates that the presence of AD dramatically changes the properties of MRI turbulence, since  $t_{\text{eddy}}$  is closely related to the turbulence autocorrelation function and the power spectrum as in Equations (10), (13), and (14). Figure 9 shows the auto-correlation function  $R(\tau) = \langle v_g(\tau)v_g(0) \rangle$  and the power spectrum for  $v_{g,x}$  and  $v_{g,z}$  in both ideal MHD

and AD runs. To derive the auto-correlation function and the power spectrum, we first output the simulation data (e.g.,  $v_{g,x}$  and  $v_{g,z}$ ) using a time interval of  $0.1\Omega^{-1}$ . We then shift the data along the  $y$  direction a distance of  $1.5\Omega x t$  to correct for the Keplerian shear. After the shift, we multiply the velocity  $v_g$  at each time frame  $t$  with the velocity at a later time  $t + \tau$ . Finally,  $R(\tau)$  is derived by averaging  $v_g(t + \tau)v_g(t)$  over both space and time. The autocorrelation functions for both vertical and radial velocity, i.e.,  $R_{zz}(\tau)/R_{zz}(0)$  and  $R_{xx}(\tau)/R_{xx}(0)$ , in both ideal MHD (VR32) and AD runs (AD1R32 and AD2R32) are shown in the upper panels of Figure 9.

The figure shows that although  $R_{zz}$  in ideal MHD (VR32) drops to zero on the timescale of  $\tau \sim \Omega^{-1}$ ,  $R_{zz}$  in AD runs (AD1R32 and AD2R32) drops off much more slowly on a timescale of  $\tau \sim 10\Omega^{-1}$ . Furthermore, in these AD runs,  $R_{zz}$  still has a positive value even at  $\tau = 100\Omega^{-1}$ . Since  $t_{\text{eddy},z} = \int_0^\infty R_{zz}(\tau)/R_{zz}(0)d\tau$ , the slow drop-off and positive tail of the correlation function in AD runs lead to the large eddy time for  $v_z$ , consistent with the direct measurement of  $t_{\text{eddy},z}$  above through tracing dust particles.

The autocorrelation function also reveals the anomalous turbulent diffusion in the  $x$  direction in AD runs. Although  $R_{xx}$  and  $R_{zz}$  are almost identical in ideal MHD runs, they have very different profiles in AD runs. In AD2R32,  $R_{xx}$  is much smaller than  $R_{zz}$ , implying  $t_{\text{eddy},x} < t_{\text{eddy},z}$ , while  $R_{xx}$  is close to  $R_{zz}$  in AD1R32 implying  $t_{\text{eddy},x} \sim t_{\text{eddy},z}$ . These relationships are consistent with the values measured in Table 3.

By doing Fourier transform for the autocorrelation functions (Equation (11)), we also compute the power

TABLE 3  
SHEARING BOX SIMULATIONS

Run name	Reso.	$X \times Y \times Z$ h×h×h	$\beta_0$	$\langle\langle v_{g,x}^2 \rangle\rangle$ ( $c_s^2$ )	$\langle\langle v_{g,y}^2 \rangle\rangle$ ( $c_s^2$ )	$\langle\langle v_{g,z}^2 \rangle\rangle$ ( $c_s^2$ )	$\alpha_R$ <sup>a</sup>	$\alpha_M$ <sup>b</sup>	$\alpha$	$\frac{d\langle x^2 \rangle}{2dt}$ ( $H^2\Omega$ )	$\frac{d\langle z^2 \rangle}{2dt}$ ( $H^2\Omega$ )	$t_{\text{eddy},x}$ $\Omega^{-1}$	$t_{\text{eddy},z}$ $\Omega^{-1}$	$Sc_x$	$Sc_z$
<b>Unstratified</b>															
VR32	32/h	1×4×2	Vert. 1e4	0.050	0.046	0.021	0.016	0.072	0.088	0.073	9.9e-3	1.5	0.47	1.2	8.9
TR32	32/h	1×4×2	Tor. 1e3	0.021	0.015	0.010	6.4e-3	0.025	0.031	0.020	9.2e-3	0.95	0.92	1.5	3.4
ZR32	32/h	1×4×2	Zero 1e4	0.013	8.1e-3	5.8e-3	3.8e-3	0.014	0.018	0.014	4.5e-3	1.1	0.78	1.3	4.0
ZR64	64/h	1×4×2	Zero 1e4	6.3e-3	4.5e-3	3.0e-3	1.7e-3	6.8e-3	8.5e-3	6.4e-3	2.8e-3	1.0	0.93	1.3	3.0
AD1R32	32/h	1×4×2	Vert. 1e3	5.7e-3	1.2e-3	4.3e-3	1.1e-3	2.9e-3	3.9e-3	0.026	0.013	4.6	3.0	0.15	0.3
AD1R64	64/h	1×4×2	Vert. 1e3	5.4e-3	1.3e-3	4.2e-3	1.0e-3	2.5e-3	3.6e-3	0.015	0.010	2.8	2.4	0.24	0.36
AD2R32	32/h	1×4×2	Vert. 1e4	8.8e-4	1.5e-4	2.7e-4	1.4e-4	3.5e-4	4.9e-4	9.3e-4	1.1e-3	1.1	4.1	0.5	0.45
AD2R64	64/h	1×4×2	Vert. 1e4	2.9e-3	5.e-4	4.4e-4	5.2e-4	4.4e-4	9.5e-4	1.3e-3	1.4e-3	0.45	3.2	0.73	0.68
AD2R128	128/h	1×4×2	Vert. 1e4	1.6e-3	3.4e-4	5.4e-4	2.4e-4	4.2e-4	6.6e-4	9.3e-4	1.4e-3	0.58	2.6	0.71	0.47
AD2R32W	32/h	8×4×2	Vert. 1e4	3.8e-3	4.1e-4	2.8e-4	5.1e-4	4.0e-4	9.0e-4	1.4e-3	8.9e-4	0.37	3.2	0.64	1.0
AD3R32	32/h	1×4×2	Vert. 2.5e4	6.8e-4	8.6e-5	8.2e-5	9.4e-5	1.4e-4	2.4e-4	4.0e-4	3.4e-4	0.59	4.1	0.60	0.71
AD3R64	64/h	1×4×2	Vert. 2.5e4	1.9e-3	3.1e-4	1.9e-4	3.3e-4	1.7e-4	5.0e-4	4.6e-4	6.0e-4	0.24	3.2	1.1	0.83
<b>Stratified</b>															
VSR32	32/h	4×4×8	Vert. 1e4												
z=[-3h,-h] <sup>c</sup>	-	-	-	0.10	0.069	0.063	0.029	0.16	0.19	0.12	0.012	1.2	0.19	1.6	16
z=[-h,h]	-	-	-	0.052	0.042	0.035	0.016	0.066	0.082	0.059	4.2e-3	1.1	0.12	1.4	20
z=[h,3h] <sup>c</sup>	-	-	-	0.12	0.086	0.087	0.033	0.18	0.21	0.078	3.6e-3	0.65	0.041	2.7	58
AD2SR32	32/h	4×4×6	Vert. 1e4												
z=[-3h,-h]	-	-	-	3.8e-3	7.3e-4	2.8e-3	5.3e-4	8.2e-4	1.4e-3	1.1e-3	2.2e-3	0.29	0.79	1.3	0.64
z=[-h,h]	-	-	-	1.1e-3	2.2e-4	8.8e-4	1.7e-4	5.4e-4	7.1e-4	9.8e-4	1.9e-3	0.89	2.2	0.72	0.37
z=[h,3h]	-	-	-	3.0e-3	3.8e-4	2.2e-3	3.8e-4	8.8e-4	1.3e-3	1.6e-3	2.8e-3	0.53	1.3	0.81	0.46
AD2SLR32	32/h	4×4×8	Vert. 1e4												
z=[-3h,-h]	-	-	-	6.9e-3	2.8e-3	5.3e-3	2.7e-3	2.9e-2	3.1e-2	5.3e-4	1.3e-3	0.077	0.25	58	24
z=[-h,h]	-	-	-	5.3e-4	1.3e-4	7.5e-4	9.5e-5	1.1e-3	1.2e-3	1.1e-3	2.6e-3	2.1	3.5	1.1	0.46
z=[h,3h]	-	-	-	3.3e-3	2.9e-4	3.8e-3	1.4e-3	3.6e-2	3.8e-3	6.3e-3	2.1e-3	1.9	0.55	0.60	1.8

<sup>a</sup> $\alpha_R \equiv \langle\langle \rho v_{g,x} \delta v_{g,y} \rangle\rangle / \langle\langle \rho \rangle\rangle c_s^2$

<sup>b</sup> $\alpha_M \equiv \langle\langle -B_x B_y \rangle\rangle / \langle\langle \rho \rangle\rangle c_s^2$

<sup>c</sup>The diffusion coefficients are measured by tracing particles from 30 to 33 orbits.

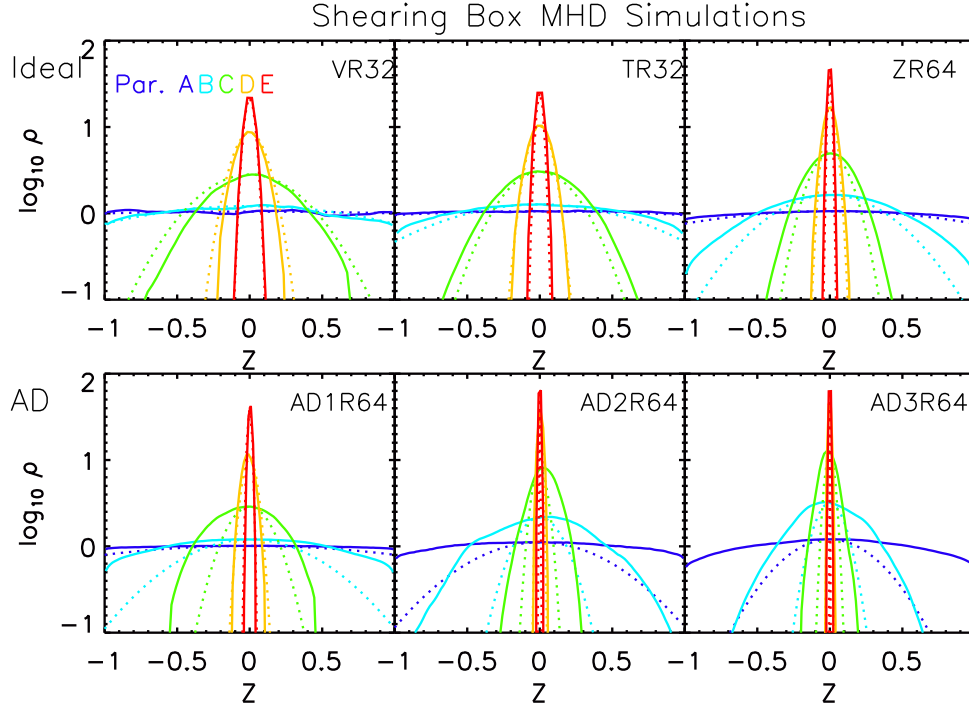


FIG. 7.— Space and time averaged density with respect to the disk height for different types of particles (colored curves) in shearing box simulations in ideal MHD (upper panels) and non-ideal MHD with AD (lower panels). The dotted curves are from the Gaussian profiles using  $h_d$  (Equation (16)) with  $t_{\text{eddy},z} = (2\Omega)^{-1}$ . As in Figure 6, the agreement is only good for ideal MHD runs.

spectrum of turbulence ( $\hat{E}_g(\omega)$ ) in both ideal and non-ideal MHD, shown in the bottom panels of Figure 9. We can see that, as  $\omega \rightarrow 0$ ,  $\pi \hat{E}_{g,z}(\omega) / \langle v_{g,z}^2 \rangle$  in AD runs be-

comes  $\sim 10\Omega^{-1}$  which is significantly larger than  $\sim \Omega^{-1}$  in the ideal MHD run. Since  $t_{\text{eddy},z} = \pi \hat{E}_{g,z}(0) / \langle v_{g,z}^2 \rangle$  (Equation (14)), this again implies a bigger  $t_{\text{eddy},z}$  in

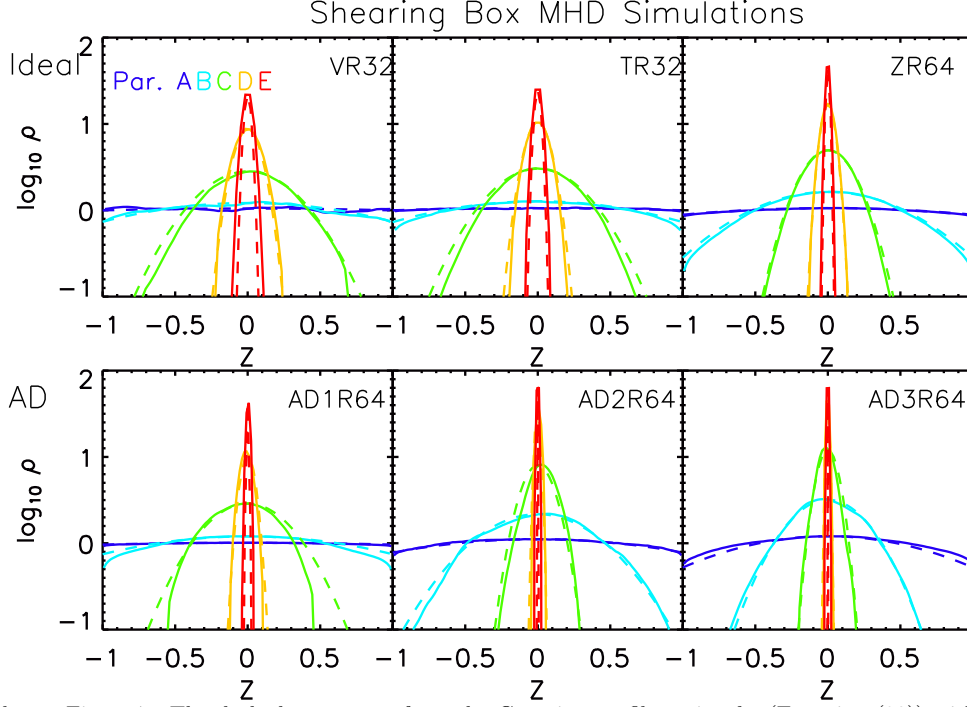


FIG. 8.— Similar to Figure 7. The dashed curves are from the Gaussian profiles using  $h_d$  (Equation (20)) with the directly measured  $t_{\text{eddy},z}$  in Table 3. The agreement is good for all the runs.

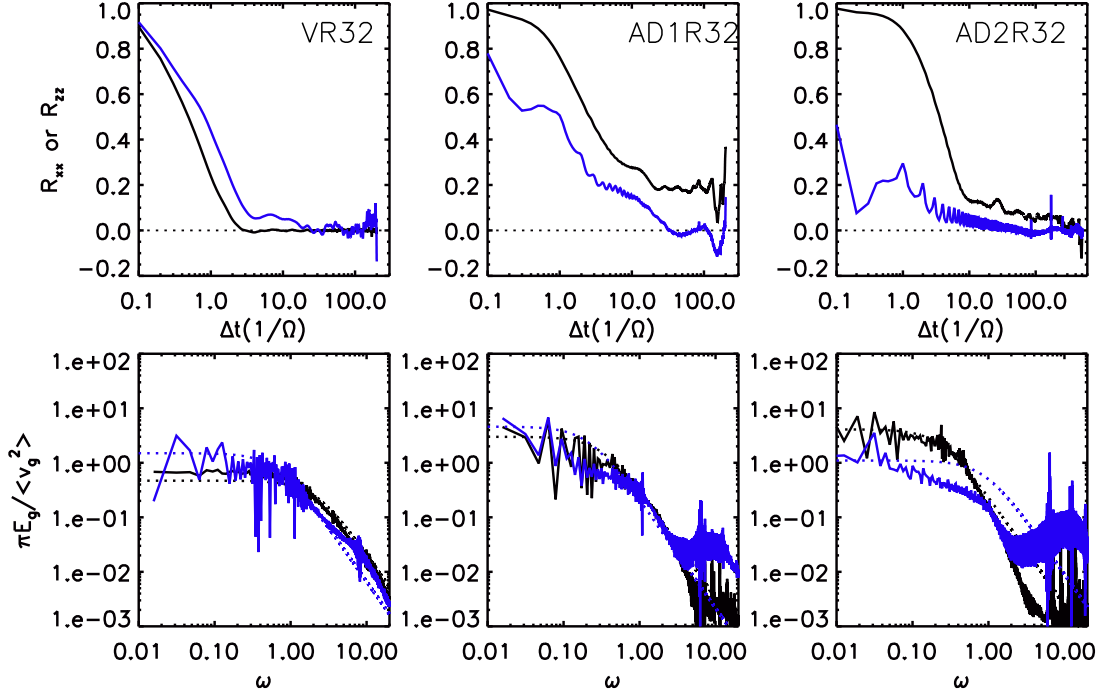


FIG. 9.— Upper panels: the auto-correlation function  $R_{xx}(\tau)/R_{xx}(0)$  (blue curves),  $R_{zz}(\tau)/R_{zz}(0)$  (black curves) for runs VR32, AD1R32, and AD2R32 (the left to right panels). AD runs have long correlation times for  $v_z$ . Lower panels: The power spectrum for  $v_x$  (blue curves) and  $v_z$  (black curves) for the same runs. The dotted curves are the analytic models from Equation (19) using  $t_{\text{eddy},x}$  and  $t_{\text{eddy},z}$  in Table 3.

AD runs.

For big particles with  $T_s > 1$ , not only  $\hat{E}_g(0)$  (or  $t_{\text{eddy}}$ ) but also the whole  $\hat{E}_g(\omega)$  over  $\omega$  determines the dust

diffusion coefficients  $D_{d,x}$  and  $D_{d,z}$ . This is because, to derive  $D_{d,x}$  and  $h_d$  in Equations (22) and (20) for big particles, we need to integrate the velocity and the power

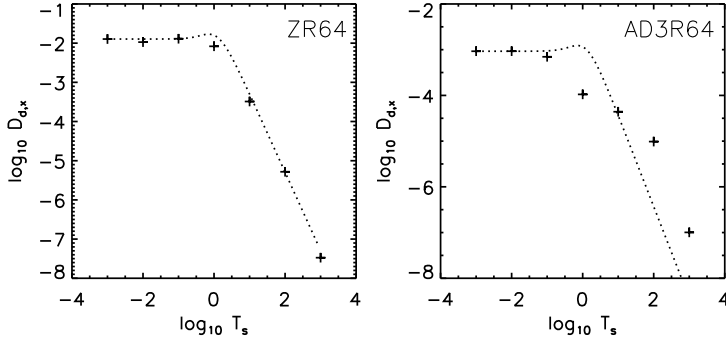


FIG. 10.— Particle radial diffusion coefficients as a function of particle stopping time, for ZR64 (the left panel) and AD3R64 (the right panel). The analytical model of YL (Equation (23)) is shown as dotted curves which agree with ideal MHD simulations very well but not completely agree with AD runs.

spectrum ( $\hat{E}_g$ , Equation (19)) over  $\omega$ . This is different from small particles ( $T_s < 1$ ) which have the particle diffusion coefficient ( $D_d$ ) equal to the gas diffusion coefficient ( $D_g$ ) that only depends on the power spectrum  $\hat{E}_g(\omega)$  at  $\omega = 0$  (Equation (14)). Thus, we compare the analytical power spectrum of Equation (19) used in Youdin & Lithwick (2007) with the power spectrum derived from our simulations in the bottom panels of Figure 9.  $t_{\text{eddy},x}$  and  $t_{\text{eddy},z}$  in Equation (19) are given in Table 3. As shown, this analytical power spectrum (dotted curves) agrees well with the power spectrum in ideal MHD simulations. However, it deviates from the power spectrum in AD runs significantly. The turnover between the integral scale and the inertial range, and the slope in the inertial range are all different from the simple analytical formula. Moreover, the power spectrum in AD simulations actually increases at the shortest timescales (largest  $\omega$ ).

Thus, this non-standard power spectrum in AD dominated disks implies that Equation (20) for  $h_d$  and Equation (22) for  $D_{d,x}$  derived using the standard power spectrum (Equation (19)) are not applicable for describing diffusion of big particles in AD disks. To illustrate this point, Figure 10 compares the dust radial diffusion coefficients measured from simulations with the analytical expression of Equation (23). As expected, the analytical theory reproduces the radial diffusion coefficients in ideal MHD runs, but fails to reproduce the coefficients at  $T_s > 1$  in AD runs. Unfortunately, we cannot test  $h_d$  (Equation (20)) for big particles with  $T_s > 1$ , since it requires a much higher vertical resolution in simulations to resolve the scale height for these particles.

The long eddy time of  $v_z$  in AD runs should also manifest itself in physical space. We average  $v_z$  in both  $y$  and  $z$  directions to derive  $\langle v_z \rangle$ , and plot its space time diagram along the  $x$  direction in Figure 11. We also include one simulation in Bai & Stone (2011) (denoted as ADR64BS) which has  $\text{Am}=1$  and  $\beta_0=1.e3$ , similar to our AD1R64 run. In the ideal MHD run (VR32),  $v_z$  has a coherence time  $\sim \Omega^{-1}$ . But in AD runs, coherent structure in  $v_z$  can exist for 100 orbits with a typical width of  $h$  in the  $x$  direction. On the other hand, Table 3 and Figure 9 suggest that, in AD runs,  $v_x$  should have a different coherent time than  $v_z$ . The space time diagrams for  $\Sigma$ ,  $\langle v_x \rangle$ ,  $\langle \delta v_y \rangle$ , and  $\langle v_z \rangle$  of AD2R32W are shown in

Figure 12.  $\delta v_y$  is  $v_y - v_{\text{kep}}$  where  $v_{\text{kep}} = -1.5\Omega x$ . The short correlation time of  $v_x$  and the long correlation time of  $v_z$  are apparent in this figure, which is consistent with  $t_{\text{eddy},x} < t_{\text{eddy},z}$  for this run in Table 3.

The long-lived structure of  $v_z$  in AD runs shown in Figure 11 and 12 implies that the linear growing mode still persists in the nonlinear phase, and such mode significantly affects particle dynamics. But the detailed mechanism deserves further studies in future.

## 5.2. Stratified Simulations

It is possible that the long-lived structure in  $v_z$  shown in Figure 11 is an artifact of the unstratified shearing box with periodic boundary conditions in the  $z$  direction. To study if our conclusions on particle settling will be changed with stratification in the gas, we have carried out stratified shearing box simulations. The gas disk has an initial Gaussian density profile in the vertical direction. The simulation domain for most simulations is  $[-2h, 2h] \times [-2h, 2h] \times [-4h, 4h]$  in the  $x$ ,  $y$ , and  $z$  directions. The boundary condition in the  $z$  direction is the same as Simon et al. (2013) which extrapolate both density and magnetic fields from the last active zone to the ghost zones. The grid resolution is 32 cells per  $h$ . The disk is threaded by a net vertical field having  $\beta_0 = 10^4$ . For each type of particles, there are  $10^6$  particles uniformly distributed in the box. Other simulation parameters and diagnostics are the same as in our unstratified simulations, unless they are specified below.

Three simulations in ideal or non-ideal MHD with AD have been carried out: (1) The ideal MHD simulation (VSR32); (2) The AD dominated simulation having  $\text{Am}=1$  in the whole box (AD2SR32); (3) The layered disk simulation (AD2SLR32) with

$$\text{Am} = \text{Am}_0 \times \left( e^{\Sigma_c/\Sigma_+} + e^{\Sigma_c/\Sigma_-} \right) \quad (32)$$

where  $\Sigma_+$  and  $\Sigma_-$  are the integrated surface density above and below each grid cell (i.e.,  $\Sigma_+ = \int_z^\infty \rho dz$ ,  $\Sigma_- = \int_{-\infty}^z \rho dz$ ,  $\Sigma_c = 0.02\Sigma$ , and  $\text{Am}_0=0.5$ ). The layered disk setup is to simulate the minimum-mass solar nebular (MMSN) at 30 AU ( $\Sigma = 10 \text{ g cm}^{-2}$ ) with an active layer of  $\Sigma_c = 0.2 \text{ g cm}^{-2}$  ionized by FUV (Bai 2014).

We divide the disks into the disk midplane within  $z = h$  and the disk atmosphere from  $h$  to  $3h$ . To measure diffusion coefficients in each region, we trace all particles that stay in that region from 30 to 40 orbits, and use Equations (9) and (21) to derive the diffusion coefficients. Mean squared velocities and stresses are measured in the same way as in unstratified simulations. All these quantities in different regions are given in Table 3.

At the disk midplane within  $z = h$ ,  $t_{\text{eddy},z}$  is again larger than one in the stratified AD simulations ( $t_{\text{eddy},z} = 2.2$  in AD2SR32, and  $t_{\text{eddy},z} = 3.5$  in AD2SLR32). Since the vertical gravity is almost zero at the disk midplane, the turbulence at the disk midplane of stratified disks should be similar to that in unstratified disks unless there is a feedback loop between the disk midplane and the disk atmosphere. Indeed, the measured eddy time at the disk midplane of these stratified simulations are consistent with those in unstratified simulations.

Despite  $t_{\text{eddy},z}$  at the midplane of AD runs is larger than 1,  $t_{\text{eddy},z}$  at the disk atmosphere is smaller than 1 in



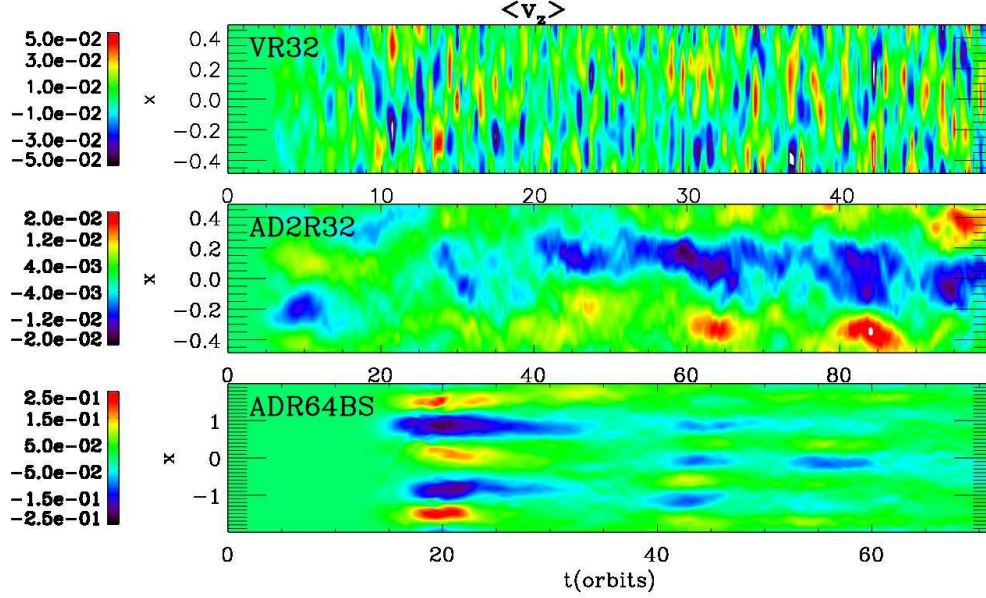


FIG. 11.— Space time diagram of  $\langle v_z \rangle$  in the  $x$  direction for various runs.  $\langle v_z \rangle$  is derived by averaging  $v_z$  in both  $y$  and  $z$  directions. In the ideal MHD run VR32,  $v_z$  has a correlation time  $\sim \Omega^{-1}$ . In AD runs, the coherent  $v_z$  forms horizontal bands in the diagram.

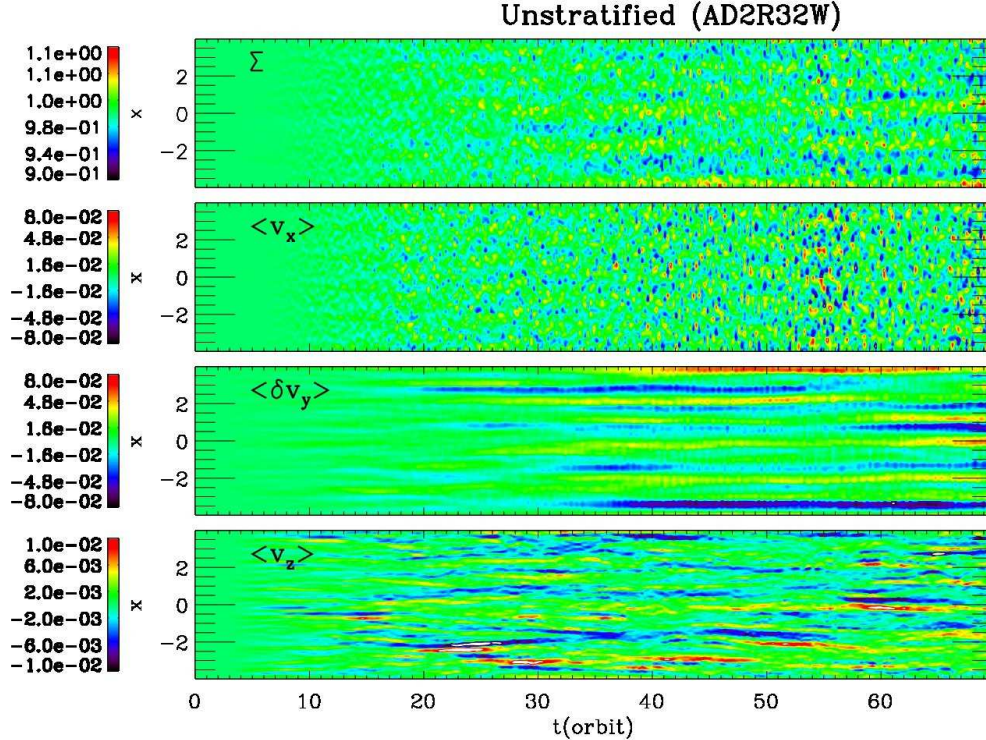


FIG. 12.— Space time diagram of  $\Sigma$ ,  $\langle v_x \rangle$ ,  $\langle \delta v_y \rangle$ , and  $\langle v_z \rangle$  in the  $x$  direction for run AD2R32W. The weak zonal flows shown in the  $\Sigma$  panel are more apparent in the  $\langle \delta v_y \rangle$  panel.  $\langle v_z \rangle$  has a longer correlation time than  $\langle v_x \rangle$ .

these disks. Considering AD2SR32 and AD2SLR32 have totally different properties in their atmospheres ( $Am=1$  for AD2SR32 and ideal MHD for AD2SLR32), the similarity in  $t_{eddy,z}$  at the atmosphere implies that  $t_{eddy,z}$  at the atmosphere is determined by turbulence generated at the midplane of these disks. We suspect that the coherent disturbance coming from the disk midplane cannot maintain its structure when it is propagating into the low density disk atmosphere, and the turbulence at the disk

atmosphere is modulated by the disk epicyclic motion or vertical oscillation.

The vertical density profiles for Par. a and b in run AD2SR32 are given in Figure 14. The solid curves are from simulations while the dotted and dashed curves are derived from Equation (15) using the midplane  $\langle v_z^2 \rangle$ . The dashed curve assumes  $t_{eddy,z} = 3\Omega^{-1}$ , while the dotted curve assumes  $t_{eddy,z} = (2\Omega)^{-1}$ . Clearly the large  $t_{eddy,z}$  leads to a better fit to the density profiles in the stratified

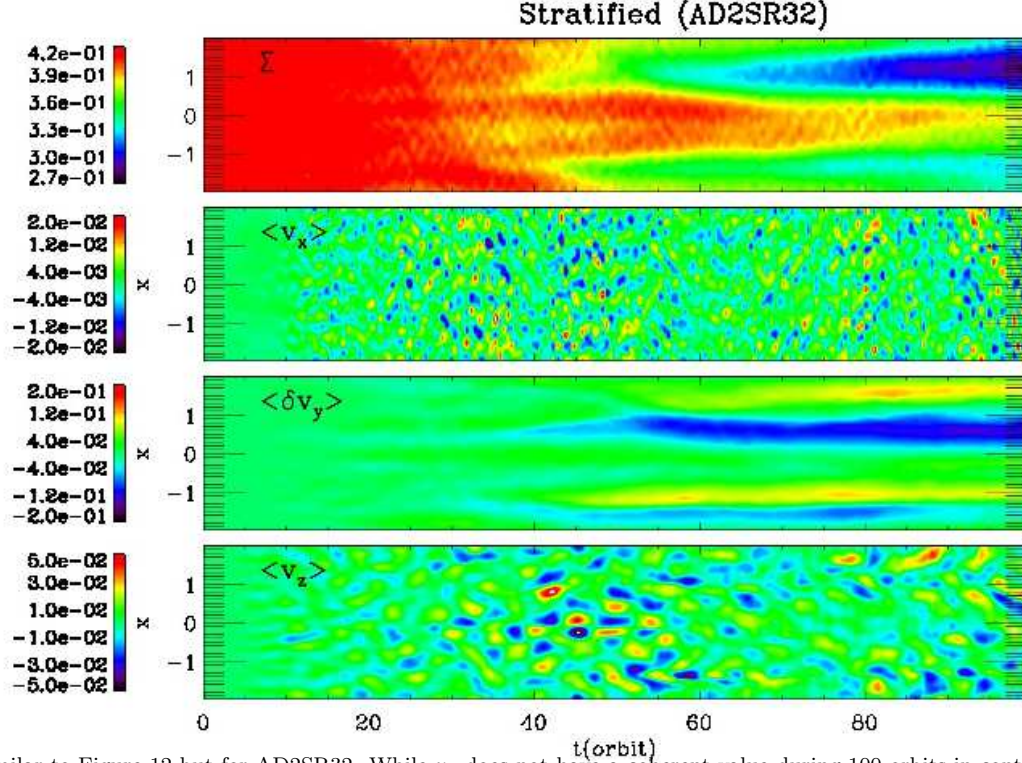


FIG. 13.— Similar to Figure 12 but for AD2SR32. While  $v_z$  does not have a coherent value during 100 orbits in contrast to unstratified simulations, it still has a longer correlation time than  $v_x$ , implying  $t_{\text{eddy},z} > t_{\text{eddy},x}$ .

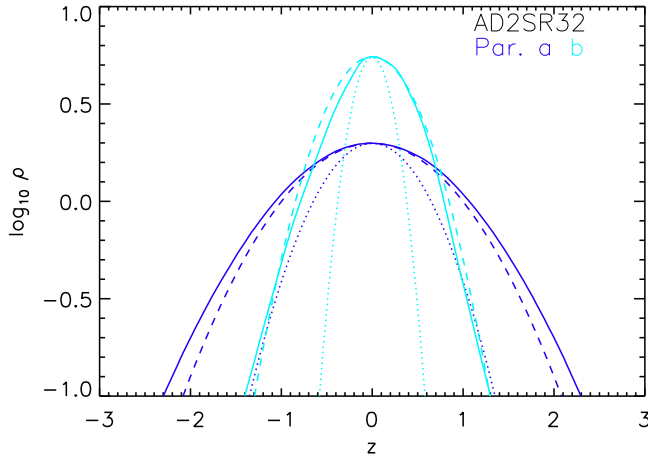


FIG. 14.— Vertical density profiles for Par. a and b in the stratified simulation. The solid curves are from simulations while the dotted and dashed curves are derived from  $h_d$  in Equation (15) using the midplane  $\langle v_z^2 \rangle$ . The dashed curve assumes  $t_{\text{eddy},z} = 3\Omega^{-1}$ , while the dotted curve assumes  $t_{\text{eddy},z} = (2\Omega)^{-1}$ . A large  $t_{\text{eddy},z}$  fits the density profiles in simulations better.

simulations.

To show the large eddy time in stratified AD simulations (e.g., run AD2SR32), we plot the space time diagram for the surface density and mean velocities along the  $x$  direction, as in Figure 13. The velocities are averaged over  $y$ , and  $[-h, h]$  in  $z$ . While  $v_z$  does not have a coherent value during 100 orbits in contrast to the unstratified simulations (Figure 12.), it still has a longer correlation time than  $v_x$ , implying  $t_{\text{eddy},z} > t_{\text{eddy},x}$ . Another noticeable feature in Figure 13 is the large-scale zonal flows in both density and  $v_y$  panels, which will be

discussed in §6.2.

## 6. DISCUSSION

### 6.1. The Eddy Time and Schmidt Number

The eddy time, also called the integral time, is one of the most important parameters to quantify turbulence, and it directly relates to turbulent diffusion. Figure 15 summarizes  $t_{\text{eddy}}$  in all our simulations. For turbulence driven by the MRI with ideal MHD, both  $t_{\text{eddy},x}$  and  $t_{\text{eddy},z} \sim \Omega^{-1}$ .  $t_{\text{eddy},z}$  can become smaller than  $\Omega^{-1}$  only when the disk is threaded by a strong net vertical magnetic field. For turbulence driven by the MRI with AD,  $t_{\text{eddy},x}$  can range from  $5\Omega^{-1}$  to  $0.2\Omega^{-1}$  as the net magnetic field strength decreases.  $t_{\text{eddy},z}$  is close to  $3\Omega^{-1}$  in both unstratified and stratified simulations.

Another important parameter to quantify turbulent diffusion is the Schmidt number ( $Sc$ ) which is normally defined as the ratio between the rates of (angular) momentum transport and mass diffusion. We want to caution that there are a few different definitions of Schmidt number in the literature: Cuzzi et al. (1993) and Youdin & Lithwick (2007) define Schmidt number as the ratio between the gas and dust diffusion coefficient. In fluid dynamics the Schmidt number is defined as the ratio of the viscous (momentum) diffusion to mass diffusion. While in turbulent disks, the Schmidt number is normally defined as the ratio between the  $R\phi$  stress (determining angular momentum transport) and dust diffusion coefficient (Johansen & Klahr 2005; Carballido et al. 2005, 2011). In this work we follow the last definition which has been widely used in numerical simulations to study dust diffusion. We only show the Schmidt number for gas fluid or well coupled particles. For particles that are less coupled to the gas, the relationship between their dust



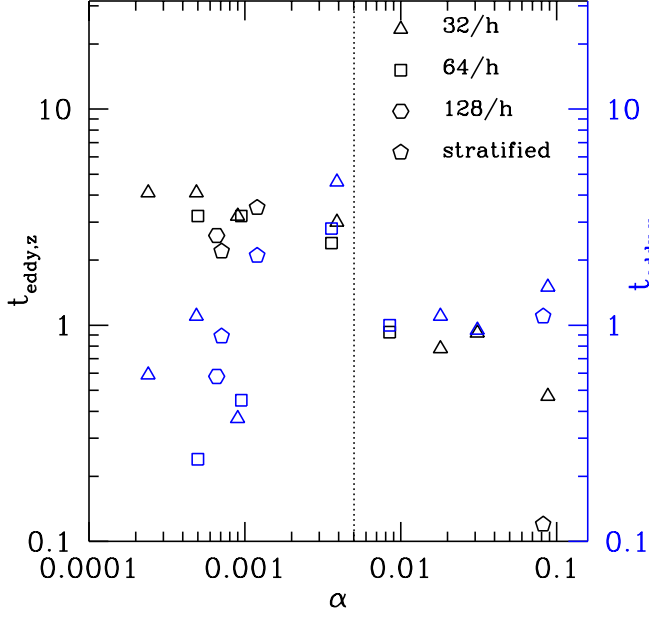


FIG. 15.—  $t_{\text{eddy},x}$  (blue points) and  $t_{\text{eddy},z}$  (black points) for all our shearing box simulations. The dotted line separates ideal MHD simulations (on the right) from MHD simulations with AD (on the left). Clearly,  $t_{\text{eddy},z} \sim 3$  for AD runs, while both  $t_{\text{eddy},x}$  and  $t_{\text{eddy},z} \sim 1$  for ideal MHD runs.

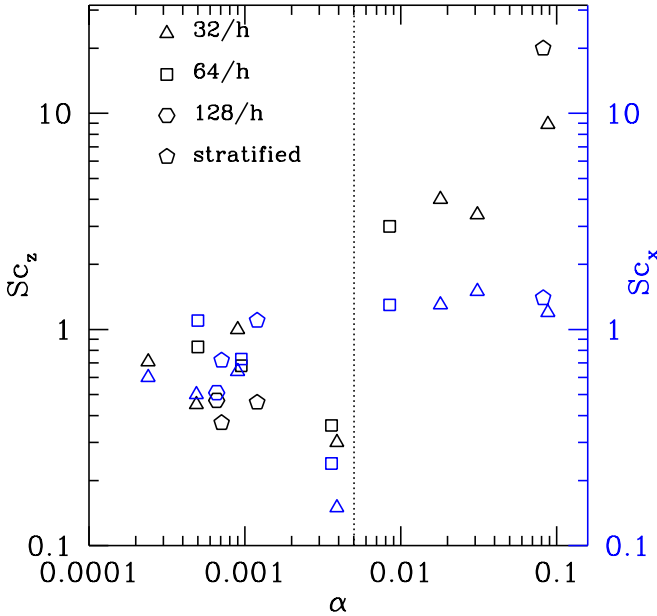


FIG. 16.—  $Sc_x$  (blue points) and  $Sc_z$  (black points) for all our shearing box simulations. The dotted line separates ideal MHD simulations (on the right) from MHD simulations with AD (on the left). Clearly,  $Sc_z$  is smaller than 1 for AD runs, while  $Sc_z \gtrsim 3$  for ideal MHD runs.

diffusion coefficients and gas diffusion coefficient has been shown in Figure 10.

We calculate  $Sc_x \equiv \alpha H^2 \Omega / D_{g,x}$  and  $Sc_z \equiv \alpha H^2 \Omega / D_{g,z}$  shown in Table 3 and Figure 16.

In ideal MHD runs,  $Sc_x \sim 1$  and  $Sc_z \gtrsim 3$ , while in AD runs, both  $Sc_x$  and  $Sc_z$  are  $\lesssim 1$ . This again suggests that there is a qualitative difference between turbulence in ideal and non-ideal MHD with AD.

## 6.2. Particle Trapping In zonal flows

Zonal flows (ZF) are axisymmetric density structures that extend over a large radial range in MRI turbulent disks. Due to the presence of these density structures, the azimuthal velocity in disks starts to deviate from a Keplerian rotation profile (e.g.,  $\delta v_y$  in Figures 12 and 13)<sup>7</sup>. ZF have been observed in both local shearing box (Johansen et al. 2009; Simon et al. 2012; Dittrich et al. 2013) and global simulations (e.g., Dzyurkevich et al. 2010; Flock et al. 2011; Uribe et al. 2011). Although they are more apparent in stratified disk simulations, they do exist in unstratified simulations as well (e.g., Lyra et al. 2008; Zhu et al. 2013).

To show the amplitude of the zonal flows in our simulations, we have plotted the space-time diagrams of the normalized gas surface density in Figure 17. At each time step, we fit a linear function for  $\ln \Sigma$  and  $\ln r$  for disks at  $r \in [1, 3]$  to derive the smooth background density, and then normalize the gas surface density with this background density. Figure 17 shows that zonal flows in ideal MHD runs have larger amplitude and width than zonal flows in AD runs. The amplitudes of the zonal flow ( $\delta \Sigma / \Sigma_{fit}$ ) in various cases are:  $\sim 0.1$  for V1e4 and T1e2 (having  $\alpha \sim 0.03$ ),  $\sim 0.07$  for V1e5 and T1e3 ( $\alpha \sim 0.02$ ),  $\sim 0.02$  for AD1e3 ( $\alpha \sim 2 \times 10^{-3}$ ), and  $\sim 0.005$  for AD2.5e4 ( $\alpha \sim 6 \times 10^{-4}$ ). Thus, the ratio between the ZF amplitude ( $\delta \Sigma / \Sigma_{fit}$ ) and  $\alpha$  is  $\sim 3$  for ideal MHD runs and  $\sim 10$  for AD runs.

The width of the zonal flow is 0.5-1 (5-10 disk scale heights) in ideal MHD runs, which is consistent with ideal MHD shearing box (Johansen et al. 2009; Simon et al. 2012; Dittrich et al. 2013) and global (Lyra et al. 2008) simulations. However, in AD runs, the zonal flow is significantly narrower with 0.2-0.5 (2-5 h) in AD1e3 and 0.1 (1 h) in AD1e4.

However, we caution that, based on local shearing-box simulations, ZF are quite different between unstratified and stratified disks. In unstratified disks, both local and global simulations with AD have ZF with small amplitude density fluctuations ( $< 5\%$  in Figure 12). On the other hand, when the disk is stratified, Figure 13 shows much wider and stronger ZF with an amplitude of  $\sim 30\%$  and a radial extent as large as the box size, consistent with Simon & Armitage (2014).

ZF may have a significant impact on planetesimal and planet formation since they can trap dust particles (Dittrich et al. 2013; Simon & Armitage 2014). As shown in Figure 3, for Par. d, there are significant density concentrations ( $\sim$  a factor of 3 increase of the density) occurring on the scale of  $\sim 0.3r$ .

Particle trapping by ZF in global disks is normally considered to be inefficient when the density variation associated with ZF is insufficient to overcome the background radial pressure gradient. Here we develop a slightly more quantitative estimate for particle trapping by ZF, and point out that this criterion is not accurate and particle

<sup>7</sup> Thus the term zonal flows

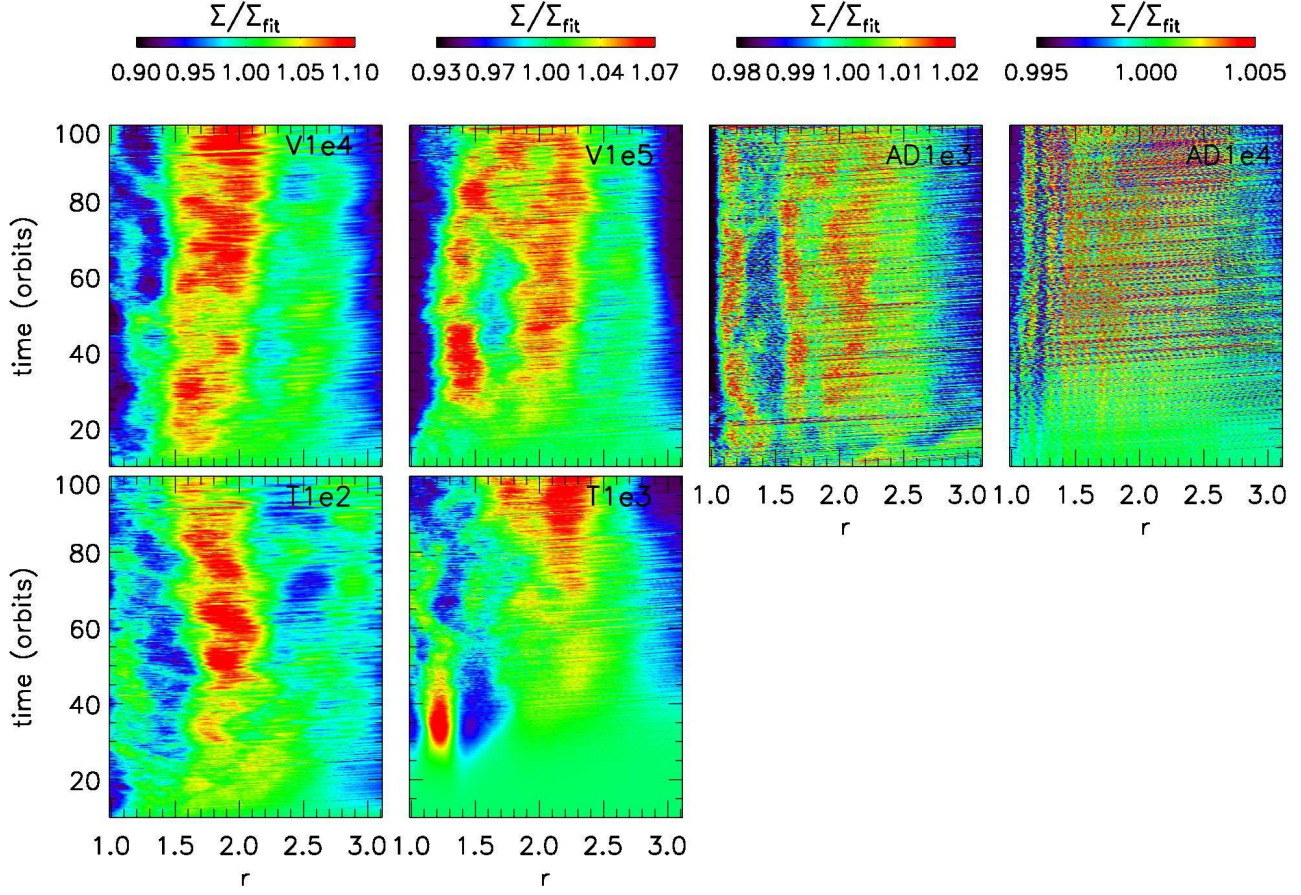


FIG. 17.— The space-time diagram of the normalized gas surface density for various cases. The strength of the disk turbulence decreases in the cases towards right. The amplitude of the zonal flow also decreases towards right.

trapping by ZF can be quite efficient even in some disks with low amplitude ZF and a large background radial pressure gradient.

As discussed in §4.1, particle trapping by ZF can be modeled using the simple 1-D particle drift diffusion equation (Equation (1)) as long as the gas surface density evolution is known from MHD simulations. This serves as a framework for our quantitative estimates on particle trapping in ZF as follows.

First, we assume that there is a gas density peak due to ZF on top of the smooth background surface density  $\Sigma_g(r)$ . The disk surface density is  $\Sigma_g(r) = \Sigma_{g,b}(r)\epsilon(r)$ , where in the peak  $\epsilon(r) > 1$ . Then, assuming the gas radial velocity is much smaller than the dust drift velocity, we have  $v_{g,r} = 0$ , and Equation (3) can be written as

$$v_{d,r} = \frac{v_K \frac{c_s^2}{r\Omega^2} \frac{\partial \ln(\Sigma_{g,b} c_s^2)}{\partial r} \left(1 + \frac{\partial \ln \epsilon / \partial \ln r}{\partial \ln \Sigma_{g,b} c_s^2 / \partial \ln r}\right)}{T_s + T_s^{-1}}. \quad (33)$$

If we denote the drift speed due to the background pressure gradient as  $v_{d,r,b}$ , we have

$$v_{d,r} = v_{d,r,b} \left(1 + \frac{\partial \ln \epsilon / \partial \ln r}{\partial \ln \Sigma_{g,b} c_s^2 / \partial \ln r}\right). \quad (34)$$

Since the radial profiles of the disk's background density and temperature normally follow power laws,  $v_{d,r,b}$  can

also be written as a power law ( $v_{d,r,b,0}(r/r_0)^\gamma$ ). Combining Equation (34) and Equation (1), assuming dust diffusion is negligible, and normalizing the equation with drift timescale  $-r_0/v_{d,r,b,0}$ , we find

$$\frac{\partial \Sigma_d}{\partial \bar{t}} - \frac{r_0}{r} \frac{\partial}{\partial r/r_0} \left[ \frac{r}{r_0} \Sigma_d \left( \frac{r}{r_0} \right)^\gamma \left( 1 + \frac{\partial \ln \epsilon / \partial \ln r}{\partial \ln \Sigma_{g,b} c_s^2 / \partial \ln r} \right) \right] = 0. \quad (35)$$

where  $\bar{t} = -tv_{d,r,b,0}/r_0$  is the time normalized by the drift timescale. From this equation, we can see that, if  $\gamma$  is a constant, dust with different sizes follows the same surface density evolution with respect to the normalized time  $\bar{t}$ , independent on  $v_{d,r,b,0}$  and the particle size.

Without considering ZF ( $\epsilon = 1$ ), Equation (35) can be solved analytically using the method of characteristics (similar to Youdin & Shu 2002). With our initial condition of  $\Sigma_d = \Sigma_{d,0}(r/r_0)^{-1}$ , the solution is

$$\Sigma_d(r, \bar{t}) = \Sigma_{d,0} \left( \frac{r_0}{r} \right)^{1+\gamma} \left( \left( \frac{r}{r_0} \right)^{1-\gamma} + (1-\gamma)\bar{t} \right)^{\gamma/(1-\gamma)}. \quad (36)$$

With our assumed disk structure and Equation (3), particles with  $T_s < 1$  have  $\gamma = -1/4$  while particles with  $T_s > 1$  have  $\gamma = 1/4$ . Thus, at the drift timescale of  $t = -r_0/v_{d,r,b,0}$  or  $\bar{t} = 1$ , we have  $\Sigma_d(r_0, 1) = 0.85\Sigma_{d,0}$  for small particles having  $\gamma = -1/4$ , and  $\Sigma_d(r_0, 1) =$

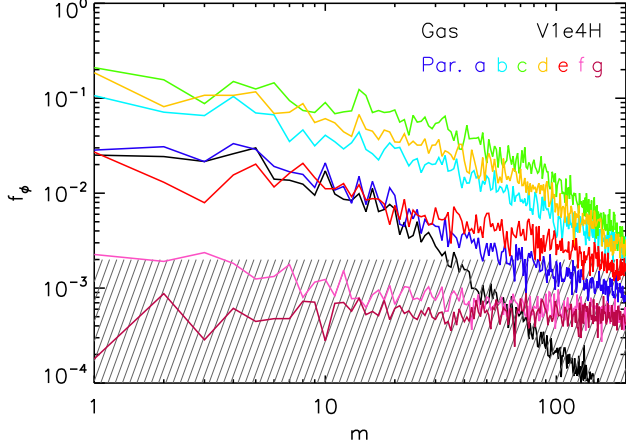


FIG. 18.— Spatial power spectrum of the disk surface density in the azimuthal direction. The black curve is for the gas surface density, while the colored curves are for the dust surface density. With increasing particle size from Par. a to Par. c, the power spectrum increases, indicating dust clumping is more significant when  $T_s$  is closer to 1. The power spectrum of very large particles (Par. e) can be even lower than the power spectrum of the gas disk. The shaded region is dominated by the Poisson noise from limited number of particles in each grid cell.

$1.21\Sigma_{d,0}$  for big particles having  $\gamma = 1/4$ . Thus, the surface density of dust changes less than 20% within the particle drift timescale in our setup. On long timescales,  $\Sigma_d$  changes at the rate of  $t^{-1/5}$  and  $t^{1/3}$  respectively, indicating that dust accumulation or depletion from the global drift is inefficient in this case.

Under the condition that the background pressure gradient leads to little dust surface density change (i.e.,  $\gamma < 1/2$ ), shallow ZF, which cause  $\gamma$  deviate from 0 within ZF, could lead to a significant particle concentration. With out unstratified disk setup, the background pressure gradient can lead to fast particle drift but won't change the disk surface density significantly ( $\gamma = 1/4$  or  $1/2$  in Equation 36). In this case, pressure bumps in our simulations can lead to significant particle concentration. As shown in Figure 3, ZF in our ideal MHD simulations can lead to a factor of  $\sim 2$ -3 dust density enhancement at density peaks for Par. d during Par. d's drift timescale ( $\sim 100$  orbits). Based on our dimensionless equation (Equation 35), if the zonal flow can last for  $\sim 1000$  orbits, Par. e could also be concentrated by a factor of  $\sim 2$ -3. Thus, particle trapping by small amplitude ZF can be efficient, as long as ZF can persist over the particle drift timescale and the background global pressure gradient leads to little dust surface density change.

On the other hand, if  $\gamma > 1/2$ , global particle drift itself can lead to large dust surface density change. Then the dust surface density change caused by weak zonal flows is not comparable to the change due to background pressure gradient, and particle trapping by pressure bumps is less significant.

### 6.3. Particle Clustering

Particles drift not only radially due to axisymmetric zonal flows but also azimuthally in response to non-axisymmetric gas structures induced by MRI turbulence. In turbulent disks, the gas density has non-zero Fourier components in the  $\phi$  direction in all  $m$ -modes. In or-

der to study how particles respond to these azimuthal gas fluctuations, we do Fourier transform for the surface density at  $r = 1$  in the  $\phi$  direction to get the power spectrum of the dust surface density. We want to caution that, due to the particle treatment of the dust, the accuracy of the dust surface density is limited by the Poisson noise of the number of particles within each grid cell. Thus, we use simulation V1e4H which has 0.3 billion particles for each particle type. In order to get good statistics at low  $m$ , we also average the power spectra derived at every orbit from 54 to 60 orbits. The final power spectrum ( $f_\phi$ ) for both gas and dust is shown in Figure 18 after being normalized by  $f_\phi(m = 0)$ . Even with such a large number of particles, the power spectrum below  $f_\phi \lesssim 10^{-3}$  is still dominated by Poisson noise. For example, Par. f and g have flat power spectra which are typical for Poisson noise.

Figure 18 shows that smaller  $m$  modes have higher power, and particles with different sizes have different power. Par a. couples with the gas so well that it almost has the same power spectrum as the gas (black solid curves). With increasing particle size from Par. b to Par. c, the power spectrum gets stronger, suggesting stronger particle concentration at all scales. The maximum power spectrum is achieved for particles with  $T_s \sim 1$  (Par. c). For bigger particles with  $T_s > 1$  (Par. d and e), the power spectra drop again. For the particles with  $T_s \gg 1$  (e.g., Par. e, Par. f), the power spectrum is even weaker than that of the gas, suggesting particles have little response to turbulence, and the disk is quite axisymmetric.

Another way to study particle concentration at various scales is to calculate the probability distribution function (PDF) for the dust surface density, as in Dittrich et al. (2013) and Hopkins (2013). Figure 19 shows the PDF for both gas and various types of particles in V1e4H. To compute the PDF, we first divide the surface density of both gas and dust by the initial surface density. Then we uniformly divide the range  $[0.1, 100]$  into 3000 bins, and among all the grids in the annulus between  $r=1$  and 1.5 we count the fraction of grid cells that have a relative density falling into each bin. Finally, we divide this value by the bin size, and average the resulting functions from each orbit between 54 and 60 orbits to plot Figure 19.

Particles with different sizes have different PDFs. The dotted curve in Figure 19 is the PDF for particles in the initial condition. Thus, the width of this curve represents the Poisson noise from the limited number of particles within each bin. Any feature in the PDF comparable to or narrower than this width is unreliable (thus, the PDF of Par. f and g is dominated by Poisson noise, similar to Figure 18). The PDF for the gas disk has a finite width due to turbulence. Par. a couples with the gas very well so that it has a similar PDF to the gas. Par. b has a much wider PDF, suggesting particles concentrate within turbulent eddies (large  $\Sigma$ ). The dust surface density enhancement is most significant for Par. c. For particles with  $T_s > 1$ , the PDF narrows again, suggesting the decoupling of dust from gas turbulence.

### 6.4. Numerical Convergence

To check the numerical convergence of our simulations, we have doubled the numerical resolution from 16/h to 32/h for both ideal and non-ideal MHD with AD. As shown in Table 1, doubling the resolution only



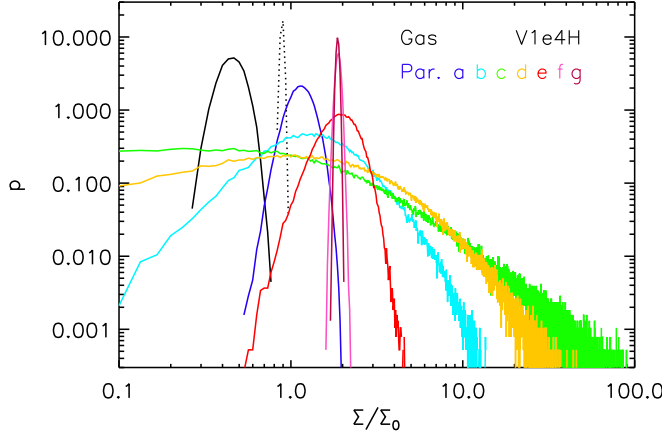


FIG. 19.— The probability distribution function (PDF) for the gas and various types of particles. The dotted curve is the PDF for the particle initial condition. The width of the dotted curve represents the Poisson noise due to the limited number of particles in each grid cell.

changes the  $\alpha$  parameter by less than 10%, and other parameters—mean squared velocities, the Reynolds and Maxwell stresses—are also similar.

The convergence of the  $\alpha$  parameter is only one diagnostic metric among many others (quality factors, spectral metrics, tilt angle) to judge the numerical convergence in global simulations (Guan et al. 2009; Noble et al. 2010; Beckwith et al. 2011; Hawley et al. 2011; Sorathia et al. 2012; Hawley et al. 2013). A thorough study on judging the convergence of global simulations by these metrics is provided in Sorathia et al. (2012) and Hawley et al. (2013). Sorathia et al. (2012), which has a similar setup as our unstratified simulations, conclude that 16 grids per scale height is required for the convergence of simulations seeded with a net field. The scale height defined in Sorathia et al. (2012) is actually  $\sqrt{2}$  of the scale height defined in this work. Thus, all our simulations fully meet this requirement.

However, we want to emphasize that, in order to have the correct global accretion structure, the simulation needs to be fully converged everywhere in the whole simulation domain. Normally, some regions in global disks are more resolved than others due to the grid spacing, and the global structure of the density, temperature and magnetic fields. For example, with uniform grid spacing and  $h \propto r^{5/4}$ , our standard cases have the resolution of  $7/h$  in the  $r$  and  $z$  directions at the inner boundary and  $90/h$  at the outer boundary. In order to quantify the resolvability at different radii in disks, we plot the quality factors  $Q$  and  $F$  as a function of  $r$  at the end of these simulations.

The quality factor  $Q$  is defined as the number of grid cells that resolve the fastest MRI growing mode (Noble et al. 2010),

$$\begin{aligned} Q_z &= \lambda_{MRI}/\Delta z, \\ Q_\phi &= \lambda_c/(r\Delta\phi), \end{aligned} \quad (37)$$

where  $\lambda_{MRI} \equiv 2\pi v_{A,z}/\Omega = 8.886\beta_z^{-1/2}h$  in ideal MHD,

and in MHD with AD

$$\lambda_{MRI} \approx 10.26 \left( 1 + \frac{1}{\text{Am}^2} + \frac{1}{\text{Am}^{1.16\epsilon}} - 0.2\epsilon \right)^{1/2} \beta_z^{-1/2} \quad (38)$$

where  $\epsilon \equiv \text{Am}/(1 + \text{Am})$  (Wardle 1999, Bai & Stone 2011). With  $\text{Am} = 1$ ,  $\lambda_{MRI} = 17.47\beta_z^{-1/2}h$ .  $\lambda_c$  is defined in the same way as  $\lambda_{MRI}$  but using  $\beta_\phi$ . The quality factor is averaged in both  $\phi$  and  $z$  directions at each  $r$ . In Figure 20, the solid black curves are  $Q_z$ , while the dotted black curves are  $Q_\phi$ . Sorathia et al. (2012) have shown that if  $Q_\phi \approx 10$ ,  $Q_z$  needs to be  $\gtrsim 10 - 15$ , and if  $Q_\phi \gtrsim 25$ ,  $Q_z$  can be smaller ( $\sim 6$  in their Figure 8). Figure 20 shows that even our standard simulations have  $Q_\phi \sim 20$  and  $Q_z \gtrsim 5$  in most regions, except those close to the inner boundary where vertical magnetic fields are lost through the inner boundary.

Another diagnostic metric proposed by Sorathia et al. (2012) is the fraction  $F$  of grids that resolve the fastest growing modes by at least eight grid cells.

$$\begin{aligned} F_z(r) &= \frac{\int (\lambda_{MRI} \geq 8\Delta z) r d\phi dz}{\int r d\phi dz}, \\ F_\phi(r) &= \frac{\int (\lambda_c \geq 8R\Delta\phi) r d\phi dz}{\int r d\phi dz}, \end{aligned} \quad (39)$$

where the logical statement within the integral takes the value of one if the statement is true, and zero if the statement is false. The  $F$  factors from our simulations are shown in Figure 20 as the blue curves. Solid blue curves are  $F_z$ , and the dotted blue curves are  $F_\phi$ . For most of our standard runs (except AD2.5e4), we have  $F_z \gtrsim 0.4$  and  $F_\phi \gtrsim 0.6$ , which means that around half of the grids have resolved the MRI fastest growing modes and the simulations should be converged (Sorathia et al. 2012). For our high resolution runs,  $F_z$  and  $F_\phi$  are larger than 0.8 in most regions, indicating most regions in our simulations have been fully resolved.

## 7. CONCLUSION

We have studied dust transport in turbulent protoplanetary disks using three-dimensional magnetohydrodynamic (MHD) simulations including Lagrangian dust particles. The turbulence is driven by the magnetorotational instability (MRI) in either ideal or non-ideal MHD with ambipolar diffusion (AD). Our aim is to test if the evolution and vertical structure of the dust disk in global 3-D MHD simulations can be reproduced by simple 1-D models. If simple 1-D models can be justified, the evolution of dust can be studied in long timescales without the need of expensive 3-D MHD simulations.

- In ideal MHD runs, we confirm that the dust radial diffusion coefficient measured in simulations agree well with the analytical formulae in Youdin & Lithwick (2007) with the assumption of  $t_{\text{eddy},x} \sim t_{\text{eddy},z} \sim \Omega^{-1}$ .
- Both the surface density evolution and vertical structure of the dust disk in 3-D global unstratified simulations can be roughly reproduced by simple azimuthally averaged 1-D and analytical models (except for dust with  $T_s \sim 1$ ).



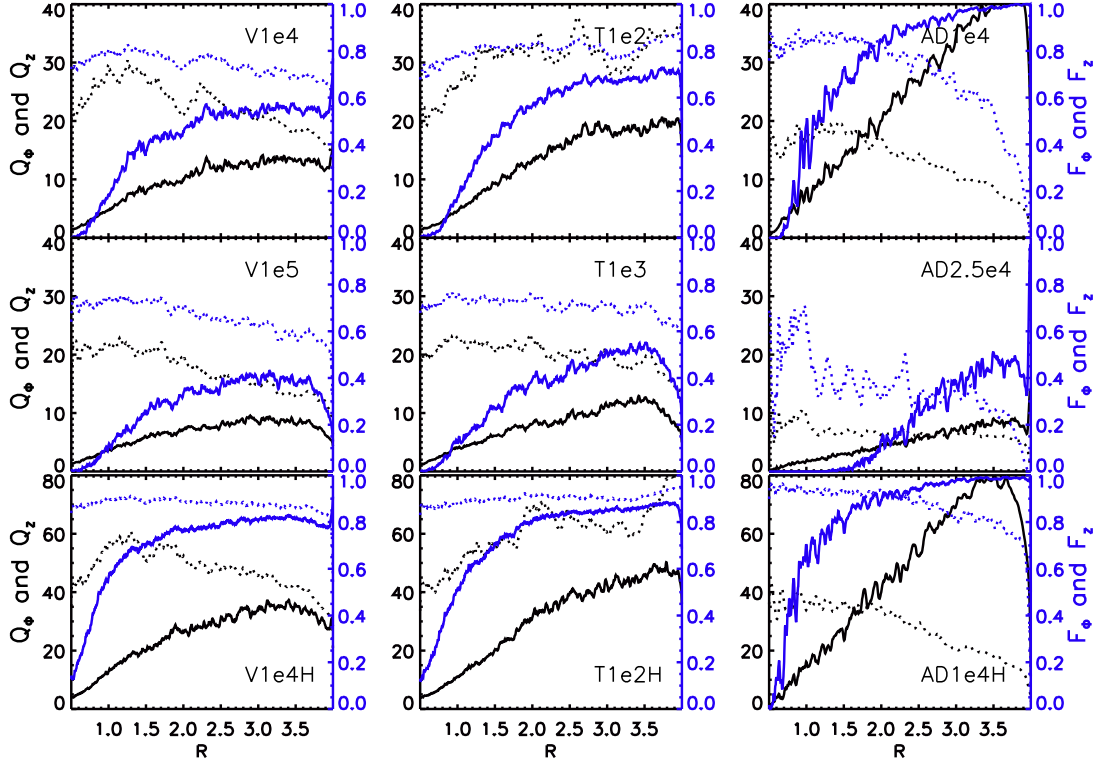


FIG. 20.— The quality factor in the  $z$  and  $\phi$  directions (Equations (37),  $Q_z$  (solid black curves),  $Q_\phi$  (dotted black curves)), and the fraction of resolved grids in the  $z$  and  $\phi$  directions (Equations (39),  $F_z$  (solid blue curves), and  $F_\phi$  (dotted blue curves)) for various runs at the end of the simulations.

- However, in order to capture particle trapping by pressure bumps due to MRI turbulence, we need a more refined 1-D model that uses the evolution of gas surface density from 3-D MHD simulations.
- There is a noticeable discrepancy between MHD simulations and azimuthally averaged 1-D models for surface density of dust having  $T_s \sim 1$ , indicating that non-axisymmetric density features in MRI turbulent disks can affect these particles significantly.
- In AD runs, turbulence is significantly suppressed, and the evolution of dust in our simulations is dominated by particle radial drift. The vertical structure of the dust disk can again be fitted by the simple analytical model but it requires a much larger  $t_{\text{eddy},z}$  than the value in ideal MHD runs.
- By carrying out both unstratified and stratified local shearing box simulations with Lagrangian particles, we find that  $t_{\text{eddy},z}$  is  $\sim 3\Omega^{-1}$  in our AD runs. On the other hand,  $t_{\text{eddy},x}$  can range from much smaller than to much larger than  $\Omega^{-1}$ , depending on the strength of the net magnetic field.
- In unstratified AD simulations, we observe that, at some parts of the disk,  $v_z$  can be even correlated over the whole simulation time. This may imply that the linear growing mode still persists in the nonlinear phase and affects particle dynamics.
- In ideal MHD runs,  $Sc_r \sim 1$  and  $Sc_z \gtrsim 3$ , implying angular momentum transport is more efficient than dust diffusion. However, in AD runs, both

$Sc_r$  and  $Sc_z$  are  $\lesssim 1$  implying dust diffusion is more efficient than angular momentum transport in the outer part of the protoplanetary disk where AD dominates.

- The difference on particle diffusion between AD and ideal MHD runs is due to that the presence of AD changes both the temporal autocorrelation function and power spectrum of turbulence. In AD runs, the temporal autocorrelation function has a long correlation time for  $v_z$ , while the correlation time for  $v_x$  can be either long or short sensitively depending on the net magnetic field strength. The temporal power spectrum also flattens or even increases at high frequencies (or short timescales). Since the detailed form of the power spectrum determines dust diffusion coefficients for particles with  $T_s \gtrsim 1$ , the formulae given by Youdin & Lithwick (2007) are not applicable for describing diffusion of big particles in AD disks.
- For global unstratified simulations, the amplitude of zonal flows becomes smaller when the turbulence becomes weaker. The weak ZF in AD runs cannot trap particles significantly. The ratio between the ZF amplitude ( $\delta\Sigma/\Sigma$ ) and  $\alpha$  is  $\sim 3$  for ideal MHD runs and  $\sim 10$  for AD runs. The width of the zonal flow is 0.5-1 (5-10 disk scale heights) in ideal MHD runs, while significantly narrower in AD runs. In our ideal MHD simulations, zonal flows can increase the dust surface density by a factor of three as long as zonal flows can persist over the particle drift timescale.

This study suggests that turbulent disks in non-ideal MHD with AD have dramatically different dust diffusion coefficients than those in ideal MHD. Future studies on particle diffusion in MRI turbulent disks dominated by non-ideal MHD effects is necessary for understanding dust transport in realistic protoplanetary disks.

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